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Elasticity
Nondestructive Testing
Lamb Waves

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INVESTIGATION OF ELASTIC WAVES IN THIN PLATES
(Unclassified Title)

by

T. N. Grigsby and
E. J. Tajchman

June 12, 1960

University of Denver, Denver Research Institute

Contract No. DA-23-072-505-ORD-1

OCO, R&D Branch Project No. _____

Department of the Army Project No. _____

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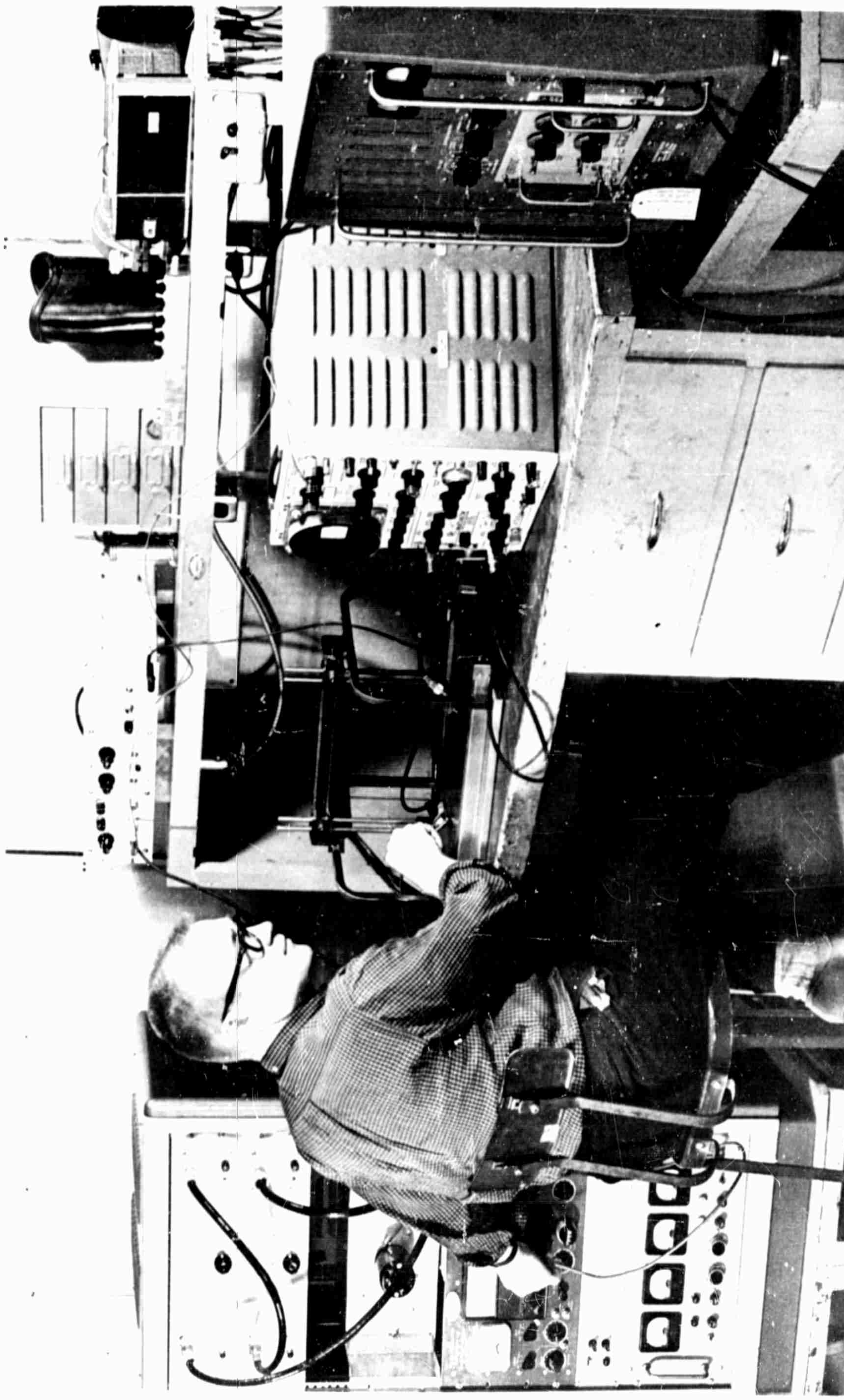

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TABLE OF CONTENTS

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ABSTRACT

A survey of the theory of Lamb waves is given. The period equation is normalized. A formula is derived for computing the group velocity from the phase velocity and the derivative of phase velocity with respect to frequency-thickness product.

Experiments are described which were conducted to determine how thin a plate must be before the modes properly called Lamb waves will be excited. Measurements of group velocity of Lamb waves are described and the values obtained are shown to agree with calculated values. Photographs of typical and atypical oscilloscope waveforms are given.

The analogy between sonic waves guided by elastic plates and electromagnetic waves is discussed.

An experiment is described using 1/32" thick steel plates, in which the insertion loss of artificial flaws was measured. The artificial flaws consisted of 1/32" wide saw-cuts of depths varying in increments of 0.003" from 0.000" to 0.021".

An experiment is described in which the insertion loss of eight different butt welds in 1/32" steel plate was measured.

An experiment is described which was performed to investigate the feasibility of a method of locating flaws in plates by the use of Lamb waves. This method showed promise.

A bibliography is given which contains one hundred and six abstracts of publications which are either related to or directly concerned with the propagation of ultrasound within plates.

INTRODUCTION

1.1 Purpose and Scope of Project

The purpose of this research project is best described by quoting from the proposal submitted to the Watertown Arsenal Laboratories by the Denver Research Institute on January 26, 1959, and entitled Technical Proposal for the Investigation of Vibrational Waves in Thin Plates:

"As technical advances are made in missile, aircraft and space vehicle design, physical reliability is becoming increasingly important. The supporting structures and covering membranes must withstand severe stresses - some of which have not yet been measured. It is obvious that the raw materials for fabricating these items must be as flawless as it is humanly possible to make them. To insure nearly absolute freedom from flaws, refined techniques of flaw detection must be kept compatible with the problems which these structures present.

"More specifically, thin sheets presently used as covering material will no doubt be made thinner as stronger materials are developed. The methods used for joining such materials, of course, will be refined continually, however, to allow the use of thin material in structures designed with marginal strength, nondestructive flaw detection methods for thin sheets must be made more precise.

"In view of these future needs, it follows that there is a continuing need for thorough understanding of all possible methods and techniques of flaw detection as well as all physical phenomena which can be used for flaw detection. It is felt that the cognizant agency involved in such testing should be also involved in the development of such refined techniques as may be required in the near future.

"It is in the area of utilization of phenomena that the present investigation is proposed. The phenomena associated with ultrasonic waves have been applied for several years in various flaw detection methods. Several methods are used for ultra-sonic flaw detection, most of them based on longitudinal, shear and surface vibrations in the material to be tested. These tests have been applied with varying degrees of success, depending upon the application. In testing thin materials or materials composed of

thin laminations, much difficulty arises due to physical dimensions and the geometry of the parts to be tested; it is difficult to get ultrasonic vibrations into thin materials and to restrain them in the material once they are generated, to say nothing of measuring these vibrations after they have traversed the area or volume to be investigated.

"There appears to be considerable hope in this type of testing for the wave motion named after Horace Lamb. Lamb[1] predicted that an infinite order of vibrational modes could be set up in a thin plate or sheet..."

The scope of this project was defined in Exhibit "A": of Contract No. DA-23-072-505-ORD-1, entitled Scope of Contract on Investigation of Vibrational Waves in Thin Plates.

GENERAL SCOPE

"Beginning with date of contract and continuing for one year, the contractor will conduct an investigation directed towards determining the feasibility of using Lamb Wave phenomena in nondestructive inspection and measuring techniques, and toward the development of such techniques to the point of practical application in sheet and plate materials such as used in supporting structures and covering membranes essential to missile, aircraft, and space vehicle design."

DETAILED SCOPE

"1. A thorough literature survey shall be conducted to reveal the state of the ultrasonic art insofar as Lamb Waves are concerned.

"2. Laboratory experimental work shall be conducted with a view to the development of practical equipment and techniques suitable for the propagation of Lamb Waves in sheet and plate metals of various dimensions and configurations.

[1]Horace Lamb, "On Waves in an Elastic Plate," Proceedings of the Royal Society of London; 506, R81, Vol. XCIII, 1917. pp 114-128.

"3. Laboratory experimental work shall be conducted with a view to the development of practical equipment and techniques suitable for the detection and measurement of Lamb Waves in sheet and plate metals of various dimensions and configurations.

"4. Laboratory experimental work shall be conducted to determine the adequacy of the above equipment and techniques for the purpose of utilizing Lamb Waves as a tool for nondestructive flaw detection and dimension measuring of sheet and plate metals of various dimensions and configurations.

"5. Experimental determinations shall be made of the sensitivity of Lamb Waves for the detection of defects of known severity and type in test pieces of various geometries made from various structural materials including but not necessarily restricted to aluminum alloys, titanium alloys and steels.

"6. An experimental evaluation shall be made of the potential possibilities of Lamb Waves as a practical tool for the field inspection of components and composite structures, in an effort to provide practical means of detecting and measuring defects therein."

1.2 Related Subject Matter

The subject matter of this project is closely related to similar work which has been done in the field of seismology. Seismologists are concerned with Lamb Waves and related phenomena not only at seismic frequencies, but at ultrasonic frequencies as well. The seismologists are interested in ultrasonic frequencies because they have found it desirable to construct small scale models and simulate earth tremors with ultrasound.

In the art of constructing ultrasonic delay lines, there is a great deal of interest in using a thin strip of metal to conduct ultrasound. This permits the construction of delay lines which have a large amount of delay within a small volume. The transmission of ultrasound through thin plates for nondestructive testing purposes is similar to the transmission of ultrasound through thin plates for the purpose of delaying a signal.

The case of waves of ultrasound being guided by a thin elastic plate is, in general, qualitatively analogous to the case of electromagnetic waves being guided by a rectangular wave guide. In the case of ultrasound being guided by a thin plate, the modes of propagation are solutions to the equations of motion subject to the boundary conditions imposed by the medium in contact with the faces of the plate. In the corresponding case of electromagnetic waves being guided by a rectangular wave guide, the modes of propagation are solutions to Maxwell's equations subject to the boundary conditions imposed by the walls of the guide.

This study has been restricted to the investigation of Lamb waves. Lamb waves include only those modes of vibration of an infinite plate in which the particle motion is independent of the direction which is parallel to the free surfaces and normal to the direction of propagation. However, there are other modes which have particle motion only in the direction which is parallel to the free surfaces and normal to the direction of propagation. These modes are called Love waves and can be interpreted in terms of totally and multiply reflected horizontally polarized shear waves. These modes are almost identical to the modes of propagation of electromagnetic waves in a rectangular wave guide.[1] It is possible that in the art of nondestructive testing there has been some application of Love waves; however, the authors of this report have uncovered no instance in which there has been a systematic attempt to develop techniques for applying them to the testing of thin plates.

The problem of the transmission of sound through panels is very closely related to the excitation of Lamb waves in plates, because sound is readily transmitted through a panel only when it is incident at such an angle that it excites the natural modes of vibration of the panel.

The problem of the vibrations of thin plates is analogous to the problems of the vibrations of prismatic bars and cylinders; because, in all three of these cases the boundary conditions at the surfaces cause the vibrations to be, generally speaking, dispersive.

1.3 Other Investigators and Their Contributions

Horace Lamb published a paper on the prolem of waves in an

[1]Ewing, W. M., Jardetzky, W. S., and Press, F., Elastic Waves in Layered Media, McGraw-Hill, New York, 1957, pp 294-295.

infinite plate in 1917.[1] His work is best described by quoting from the first page of this paper:

> "It occurred to me some time ago that a further examination of the two dimensional problem was desirable for more than one reason. In the first place, the number of cases in which the various types of vibration of a solid, none of whose dimensions is regarded as small, have been studied is so restricted that any addition to it would have some degree of interest, if merely as a contribution to elastic theory. Again, modern seismology has suggested various questions relating to waves and vibrations in an elastic stratum imagined as resting on matter of a different elasticity and density. These questions naturally present great mathematical difficulties, and it seemed unpromising to attempt any further discussion of them unless the comparatively simple problem which forms the subject of this paper should be found to admit of a practical solution. In itself it has, of course, no bearing on the questions referred to.
>
> "Even in this case, however, the period-equation is at first sight rather intractable, and it is only recently that a method of dealing with it (now pretty obvious) has suggested itself. The result is to give, I think, a fairly complete view of the more important modes of vibration of an infinite plate, together with indications as to the character of the higher modes, which are of less interest. . ."

In 1945, Firestone and Ling published a report on the use of Lamb waves in nondestructive testing.[2] They discussed possible uses of Lamb waves as well as group velocity, phase velocity, particle motion and means of generation. They made the necessary calculations and plotted curves of group velocity and phase velocity as functions of frequency-thickness product for aluminum. Firestone and Ling conducted experiments to obtain information about the nature of Lamb waves, and found evidence of mode conversion when Lamb waves were reflected at the edges of plates. They discussed the effect of the spectrum of the pulsed

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[1] Lamb, H., op. cit., p. 114

[2] Firestone, F. A., and Ling, D. S., Report on the Propagation of Waves in Plates - Lamb and Rayleigh Waves, Sperry Products, Inc., Tech. Rep. 50-6001, Danbury, Connecticut, 1945.

sine wave on the generation of Lamb waves. They experimented with various designs of transducers for exciting Lamb waves in plates; the basis for these designs was a Y-cut quartz crystal with an X dimension seven times its Y dimension and a very long Z dimension. Firestone and Ling hold United States Patent No. 2,536,128 (1951) entitled "Methods and Means of Generating and Utilizing Vibrational Waves in Plates" which discloses the curves of phase velocity versus frequency-thickness product mentioned above, methods and devices for inducing Lamb waves in plates, methods for testing plates for defects by mode conversion of Lamb waves, and a method for determining the thickness of a plate by utilizing the interference between two different modes of slightly different wavelength.

In 1953, Tolstoy and Usdin published a ray theory of the dispersive properties of stratified media.[1] They gave several examples of the use of ray theory. The propagation of Lamb waves was the simplest case considered, and they showed that the period equation obtained using ray theory was identical to the equation obtained by Lamb who started, in essence, from Hooke's law and Newton's second law of motion.

In 1955, Mindlin[2] published an extensive mathematical treatment of the vibrations of elastic plates which in general are finite, nonisotropic and have various conditions of face loading. He discussed Lamb waves as the simpler case of an isotropic, infinite, elastic plate with free faces.

In 1959, Worlton reported work conducted for the purpose of disclosing the nature of Lamb waves with a view towards applying them to nondestructive testing.[3] In addition to corroborating the results of Firestone and Ling, Worlton's experiments showed that a considerable amount of curvature of the plate did not affect the propagation of Lamb waves within it, and that the period equation as derived by Lamb for a

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[1] Tolstoy, I., and Usdin, E., "Dispersive Properties of Stratified Elastic and Liquid Media: a Ray Theory," Geophysics, v28, pp 844-870, 1953.

[2] Mindlin, R. D., An Introduction to the Mathematical Theory of Vibrations Of Elastic Plates, U.S. Army Signal Corps Engineering Laboratories, Ft. Monmouth, New Jersey, 1955, Signal Corps Contract DA-36-039-sc-56772.

[3] Worlton, D.C., Lamb Waves at Ultrasonic Frequencies, HW-60662, Hanford Atomic Products Operation, Richland, Washington, 1959.

plate in vacuum still gave satisfactory results when used to analyze the case of a plate immersed in water. Worlton manipulated the period equation to render it much more tractable for its computation on a digital computer. He used an IBM 709 digital computer to compute curves of phase velocity as a function of frequency-thickness product for aluminum, zirconium, stainless steel, brass and uranium. Worlton studied various geometries such as might be encountered in actual testing situations; these included thin strips, small diameter disks, metal overlying various diameters of flat bottomed holes, and adhesive and metallic bonds of various degrees of bond integrity.

Previously, Worlton had published work on the use of Lamb waves in practical testing problems.[1] He described the propagation of Lamb waves in a test block with a built-in laminar flaw 0.020 in below the surface, a probe for the interior inspection of tubing, and the propagation of Lamb waves in metal strip of different grain structures.

In 1948, Sauter published a paper which summarized the theory of the vibrations of thin elastic plates. In this paper, the author discussed the three possible types of vibrations: Love waves, symmetrical Lamb waves, and asymmetrical Lamb waves. He presented the period equation for each case and indicated a method of obtaining simplified equations to the first or to higher orders of approximation.[2]

1.4 Subject Matter to be Covered

The subject matter of this report parallels the detailed scope of the contract as quoted in section 1.1. One of the prime objectives of the project was to make a thorough literature search. The results of this search are presented as a survey of theory and a bibliography which includes an abstract for each reference given. In conducting this literature search, emphasis was given to articles of a basic nature, and a fairly large amount of reading was done for the purpose of acquiring background for appraising the various articles. In parallel with the

[1] Worlton, D. C., "Ultrasonic Testing with Lamb Waves," Nondestructive Testing, v 15, n 4, pp 218-222.

[2] Sauter, F., "Remarks Concerning the Theory of Vibrations of Thin Elastic Plates," Z. Natursforsch, v A3, pp 548-552, (1948) (In German)

literature search, experimental work was conducted. Some time was required to obtain the various parts of the apparatus and become familiar with it; consequently, during the earlier stages of the project, emphasis was placed on the literature search and accompanying theoretical work. During the latter stages of the project, emphasis was placed upon the experimental work.

THEORY

2.1 Analysis of the Vibrations of Infinite Plates

2.1.1 Introduction

Lamb waves are waves which can exist in an isotropic-lossless-elastic plate of finite thickness but of infinite length and width which is bounded on both of its faces by vacuum, and which are independent of a direction which is parallel to the free surfaces and normal to the direction of propagation.

Several theoretical treatments of Lamb waves may be found in the literature.[1] However, a discussion of the theory of the vibrations of infinite plates will be given here by way of summary.

The classical method of obtaining the period equation (i.e. the equation which relates phase velocity to wave length) is by making use of Hooke's law, Newton's second law of motion, and by satisfying the boundary conditions that the tensional stress normal to the free surfaces and the shear stress tangential to the free surfaces must vanish.

2.1.2 Definitions of Stress, Strain, and Rotation

Before Hooke's law and Newton's second law of motion can be stated in microscopic terms, it is necessary to define the different components of stress and strain in terms of a differential element of volume[2] as shown in Figure 1. When the solid is unstrained, the far corner of

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[1] See for example, Ewing, W.M., Jardetzky, W.S., Press, F., Elastic Waves in Layered Media, McGraw-Hill, New York, 1957, pp 281-288.

Worlton, D.C., Lamb Waves at Ultrasonic Frequencies, HW-60662 Hanford Atomic Products Operation, Richland, Washington 1959, pp 5-18.

Lamb, H., "On Waves in an Elastic Plate," Proc. Roy. Soc. (London), 93, 114-128 (1917).

[2] Kolsky, H., Stress Waves in Solids, Clarendon Press, Oxford, 1953, pp 4-7.

Mason, W.P., Piezoelectric Crystals and Their Application to Ultrasonics, D. Van Nostrand Co., New York, 1959, pp 19-29.

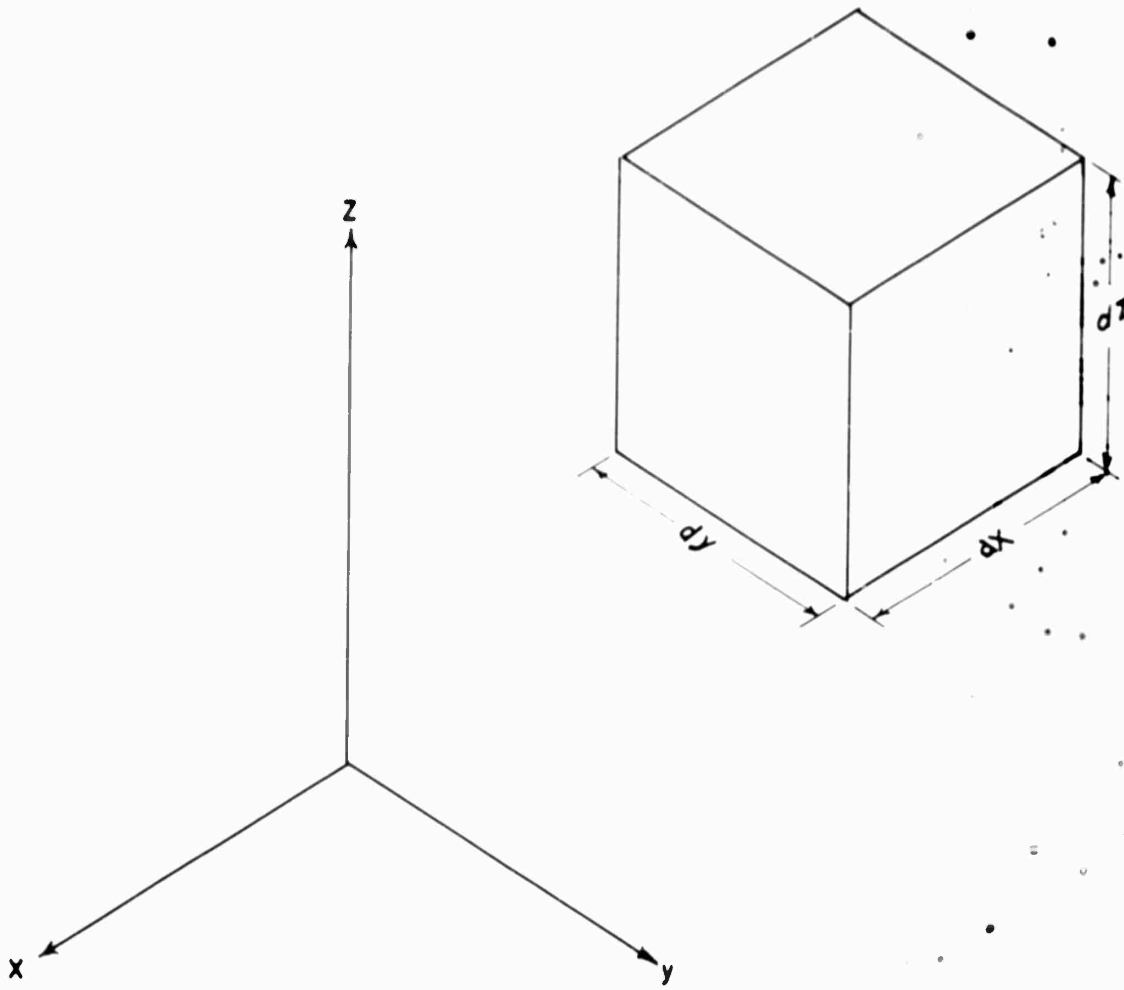

FIGURE 1

this volume element has the coordinates (x, y, z,) and its edges are of equal length, $dx = dy = dz$, as shown. The six components of stress are shown in orthographic projections of this same volume element (Figures 2 through 7). T_1, T_2, T_3, are the components of tension (compression is considered to be negative tension) parallel to the x, y, and z axes respectively. T_4, T_5, T_6, are the corresponding components of shearing stress.

The displacement of any point in a solid is resolved into the three components u, v, and w, parallel to the x, y, and z axes respectively. In other words, if the coordinates of a particle when the solid is unstrained are (x, y, z) its coordinates after being strained will be (x + u, y + v and z + w). The strain is defined in terms of the partial derivatives of the particle displacements; the three components of tensional strain are:

$$S_1 = \frac{\partial u}{\partial x}, \quad S_2 = \frac{\partial v}{\partial y}, \quad \text{and } S_3 = \frac{\partial w}{\partial z} \qquad (1)$$

The deformation and displacement of the volume element when all strains except S_1 are zero is shown in Figure 9. Figure 8 shows the same volume element when there is no strain. The dilation is the increase in volume per unit volume which is equal to $(S_1 + S_2 + S_3)$ and is denoted by Δ. The three components of shearing strain are:

$$S_4 = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \quad S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad S_6 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad (2)$$

Figure 11 shows the deformation of the volume element as a result of S_6 when all other strains are zero; note that $S_6 = \frac{\partial u}{\partial y} \cong 0$. In addition to the six components of strain, there are three components of rotation which are defined in terms of the partial derivatives of displacement as follows:

$$\overline{\omega}_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right], \quad \overline{\omega}_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right], \quad \overline{\omega}_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \qquad (3)$$

Figure 12 shows the volume element which has been rotated under a condition of zero strain. This might be the result of rotating the entire solid. Again note that $\overline{\omega}_z = -\theta$. In Figure 13 is shown the volume element which has undergone a shearing strain $S_6 = \theta$ without undergoing any rotation.

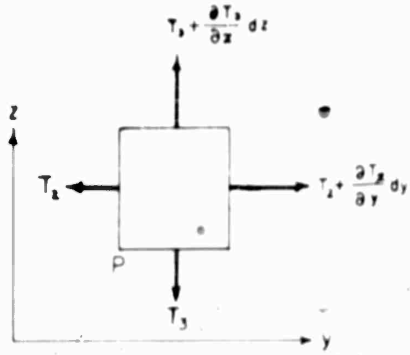

FIGURE 2

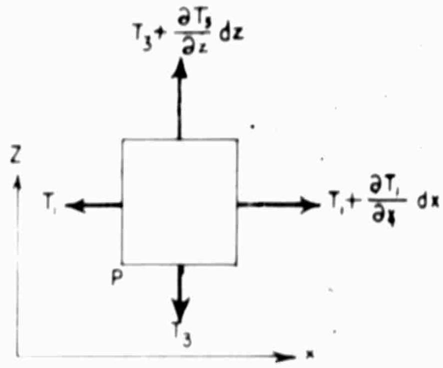

FIGURE 3

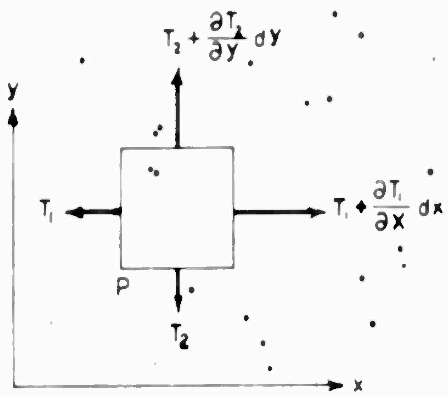

FIGURE 4

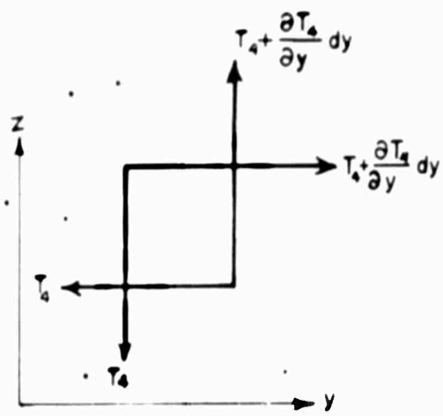

FIGURE 5

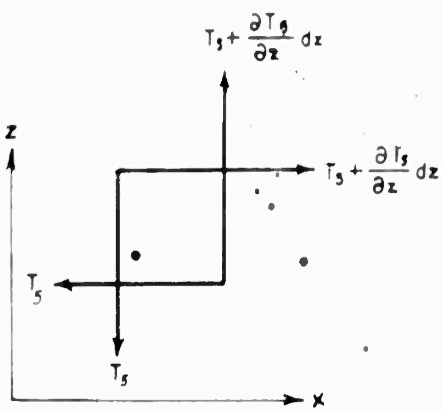

FIGURE 6

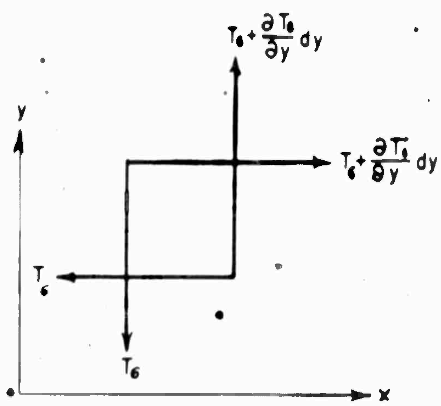

FIGURE 7

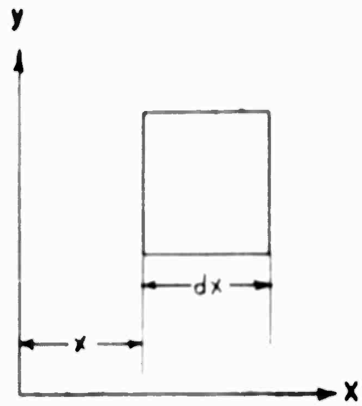

FIGURE 8

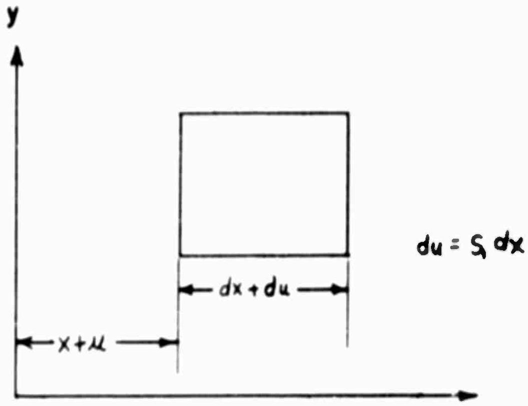

FIGURE 9

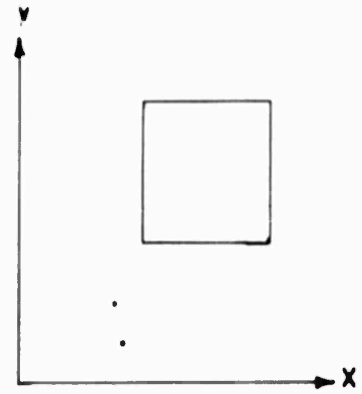

FIGURE 10

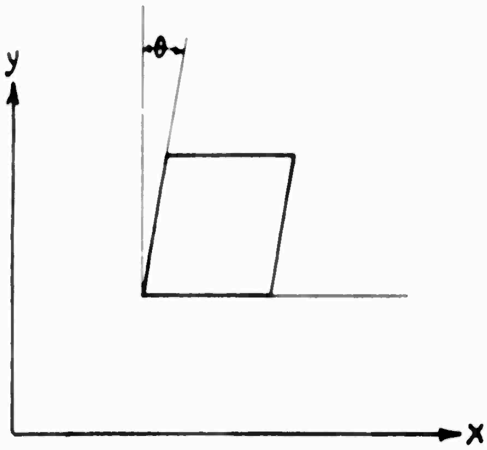

FIGURE 11

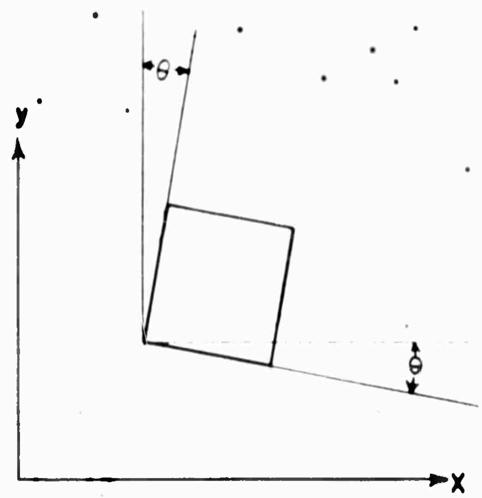

FIGURE 12

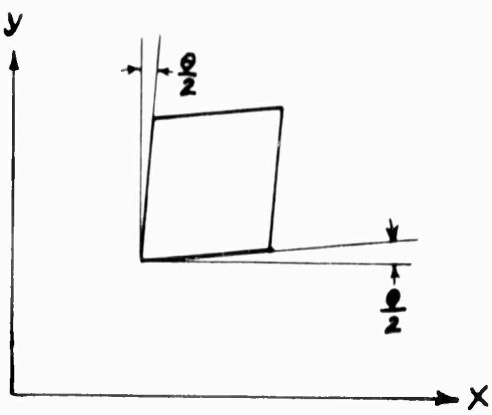

FIGURE 13

2.1.3 Hooke's Law[1]

The general form of Hooke's law is:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} \qquad (4)$$

However, in the case of an isotropic solid it can be shown, that all elastic moduli which are not zero can be expressed in terms of two constants, λ and μ; so that Hooke's law becomes:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} \qquad (5)$$

[1] Kolsky, op. cit., pp 7-10.

2.1.4 Equations of Motion[1]

The equations of motion are the result of applying Newton's second law of motion and Hooke's law to the volume element. Let F_x be the component of force (body forces such as gravity are being neglected here) parallel to the x axis; let a_x be the resulting component of acceleration and let m be the mass of the volume element. Then

$$F_x = \left[\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z}\right] dx\, dy\, dz$$

$$m = \rho\, dx\, dy\, dz \; ; \; \rho = \text{density}$$

$$a_x = \frac{\partial^2 u}{\partial t^2}$$

$$F_x = m a_x$$

$$\left[\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z}\right] dx\, dy\, dz = \rho \frac{\partial^2 u}{\partial t^2}\, dx\, dy\, dz$$

By expanding equation 5 and differentiating, one obtains:

$$\frac{\partial T_1}{\partial x} = \lambda \frac{\partial \Delta}{\partial x} + 2\mu \frac{\partial S_1}{\partial x} \quad \text{where } \Delta = (S_1 + S_2 + S_3) \text{ as defined in 2.1.2}$$

$$\frac{\partial T_6}{\partial y} = \mu \frac{\partial S_6}{\partial y}$$

$$\frac{\partial T_5}{\partial z} = \mu \frac{\partial S_5}{\partial z}.$$

By substituting S_1, S_5, and S_6 as defined in equations one and two, one obtains:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial x} + \mu \nabla^2 u, \text{ similarly,}$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial y} + \mu \nabla^2 v \qquad (6)$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial z} + \mu \nabla^2 w$$

where $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. These are called the equations of motion.

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[1]Kolsky, op. cit., pp 10-14.

2.1.5 Derivation of the Period Equation for Lamb Waves

The type of wave motion we are studying is independent of z, i.e. all cross sections of the plate parallel to the xy plane are identical, and there is no component of particle motion parallel to the z axis. Consequently, it is required to satisfy only the first two of equations 6. In this two-dimensional problem, the x axis is assumed to be horizontal with its positive direction to the right and the y axis is assumed to be vertical with its positive direction upward. The plate is assumed to be located so that it is symmetrical about the xz plane.

It is easily verified that the solution to the equations of motion, for this two-dimensional case is:

$$u = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y}; \qquad v = \frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x} \tag{7}$$

where ϕ and ψ are auxiliary functions satisfying the equations:

$$\rho\frac{\partial^2\phi}{\partial t^2} = (\lambda + 2\mu)\nabla^2\phi, \quad \rho\frac{\partial^2\psi}{\partial t^2} = \mu\nabla^2\psi \tag{8}$$

Since $\sqrt{\frac{\lambda + 2\mu}{\rho}} = V_L$ = the velocity of a longitudinal wave in an extended medium and $V_S = \sqrt{\frac{\mu}{\rho}}$ = the velocity of a shear wave in an extended medium, equations 8 may be written as follows:[1]

$$\frac{\partial^2\phi}{\partial t^2} = V_L^2\nabla^2\phi, \quad \frac{\partial^2\psi}{\partial t^2} = V_S^2\nabla^2\psi \tag{9}$$

It is assumed that there is periodicity with respect to both x and t; so that ϕ and ψ may be written,

$$\phi = F_{(y)}\, e^{i(\sigma t - \xi x)} \tag{10}$$

$$\psi = G_{(y)}\, e^{i(\sigma t - \xi x)}$$

where σ = angular frequency = $\frac{2\pi}{\text{period}} = 2\pi f$

f = frequency in cycles per second

ξ = wave number = $\frac{2\pi}{\text{wavelength}} = 2\pi/\lambda'$

λ' = wavelength

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[1] Worlton, D. C., op. cit., pp 5-8.

It is readily seen that

$$\frac{\partial^2\phi}{\partial t^2} = -\sigma^2\phi \tag{11}$$

$$\frac{\partial^2\phi}{\partial x^2} = -\xi^2\phi \tag{12}$$

so that from equation 9,

$$\frac{\partial^2\phi}{\partial y^2} = \left(\xi^2 - \frac{\sigma^2}{V_L^2}\right)\phi$$

defining $\alpha = \sqrt{\xi^2 - \frac{\sigma^2}{V_L^2}}$

$$\frac{\partial^2\phi}{\partial y^2} = \alpha^2\phi \tag{13}$$

and since $\sigma = V\xi$ where $V \equiv$ phase velocity $\equiv f\lambda'$

$$\alpha = \xi\sqrt{1 - \frac{V^2}{V_L^2}}$$

So that for a given value of phase velocity and wavelength, the manner in which ϕ varies with respect to y consists of a solution to an ordinary differential equation in y.

Similarly

$$\frac{\partial^2\psi}{\partial y^2} = \beta^2\psi \quad \text{where} \quad \beta \equiv \xi\sqrt{1 - \frac{V^2}{V_S^2}} \tag{14}$$

The solutions to equations 13 and 14 are familiar as

$$F_{(y)} = ae^{\alpha y} + be^{-\alpha y} \tag{15}$$

$$G_{(y)} = ce^{\beta y} + de^{-\beta y}$$

$F_{(y)}$ and $G_{(y)}$ may be written as hyperbolic functions so that

$$\phi = (A\sinh\alpha y + B\cosh\alpha y)\, e^{i(\sigma t - \xi x)} \tag{16}$$

$$\psi = (C\sinh\beta y + D\cosh\beta y)\, e^{i(\sigma t - \xi x)}$$

To obtain the period equation, it is necessary to satisfy the boundary conditions that T_2 and T_6 vanish at the two values of $y = \pm d/2$ where d is the thickness of the plate.

T_2 and T_6 are obtained from equation 5. (Since the motion is independent of z, S_3, S_4 and S_5 are identically zero.)

$$T_2 = \lambda \Delta + 2\mu S_2,$$

where

$$\Delta = \nabla^2 \phi, \text{ so that from equation 12 and 13,}$$

$$\Delta = (\alpha^2 - \xi^2)\phi$$

$$S_2 \equiv \frac{\partial v}{\partial y} = \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x \partial y} = \alpha^2 \phi - \frac{\partial^2 \psi}{\partial x \partial y}$$

$$T_2 = \Big[\lambda(\alpha^2 - \xi^2) A \sinh \alpha y + \lambda(\alpha^2 - \xi^2) B \cosh \alpha y + 2\mu\alpha^2 A \sinh \alpha y + 2\mu\alpha^2 B \cosh \alpha y + 2i\xi C\beta\mu \cosh \beta y + 2i\xi D\beta\mu \sinh \beta y\Big] e^{i(\sigma t - \xi x)}$$

Collecting terms and noting that

$$-\lambda\xi^2 + \lambda\alpha^2 + 2\mu\alpha^2 = \mu(\xi^2 + \beta^2),$$

$$T_2 = \Big[A\mu(\xi^2 + \beta^2)\sinh \alpha y + B\mu(\xi^2 + \beta^2)\cosh \alpha y + C(2i\xi\beta\mu)\cosh \beta y + D(2i\xi\beta\mu)\sinh \beta y\Big] e^{i(\sigma t - \xi x)} \qquad (17)$$

$$T_6 = \mu S_6$$

$$S_6 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial \psi}{\partial x^2} + \frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y^2}$$

$$T_6 = \Big[A(-2i\xi\alpha\mu)\cosh \alpha y + B(-2i\xi\alpha\mu)\sinh \alpha y + C\mu(\xi^2 + \beta^2)\sinh \beta y + D\mu(\xi^2 + \beta^2)\cosh \beta y\Big] e^{i(\sigma t - \xi x)} \qquad (18)$$

To make the manipulation less cumbersome, let

$$2i\xi\beta\mu \equiv n$$

$$-2i\xi\alpha\mu \equiv p$$

$$\mu(\xi^2 + \beta^2) \equiv q$$

$$\sinh \frac{\alpha d}{2} = s_\alpha$$

$$\sinh \frac{\beta d}{2} = s_\beta$$

$$\cosh \frac{\alpha d}{2} = c_\alpha$$

$$\cosh \frac{\beta d}{2} = c_\beta$$

Then since at $y = \pm d/2$, $T_2 = T_6 = 0$, four simultaneous equations in the four unknowns A, B, C, and D are obtained which, when written in matrix form, appear as follows:[1]

$$\begin{array}{ccccc} A & B & C & D & \\ \end{array}$$

$$\begin{bmatrix} qs_\alpha & qc_\alpha & nc_\beta & ns_\beta & 0 \\ pc_\alpha & ps_\alpha & qs_\beta & qc_\beta & 0 \\ -qs_\alpha & qc_\alpha & nc_\beta & -ns_\beta & 0 \\ pc_\alpha & -ps_\alpha & -qs_\beta & qc_\beta & 0 \end{bmatrix} \quad (19)$$

If the second and fourth columns of this matrix are interchanged, it may be manipulated so that it appears as follows:

$$\begin{array}{ccccc} A & D & C & B & \\ \end{array}$$

$$\begin{bmatrix} qs_\alpha & ns_\beta & 0 & 0 & 0 \\ pc_\alpha & qc_\beta & 0 & 0 & 0 \\ 0 & 0 & nc_\beta & qc_\alpha & 0 \\ 0 & 0 & qs_\beta & ps_\alpha & 0 \end{bmatrix} \quad (20)$$

If this system is to have any solution other than A = B = C = D = 0, it is necessary that the system determinant vanish. That is

$$\begin{vmatrix} qs_\alpha & ns_\beta \\ pc_\alpha & qc_\beta \end{vmatrix} \cdot \begin{vmatrix} nc_\beta & qc_\alpha \\ qs_\beta & ps_\alpha \end{vmatrix} = 0 \quad (21)$$

The system determinant will therefore be equal to zero if either of these second order determinants is zero; furthermore it is clear from the matrix that any solution of either of the second order systems,

$$\begin{bmatrix} qs_\alpha & ns_\beta & 0 \\ pc_\alpha & qc_\beta & 0 \end{bmatrix}, \qquad \begin{bmatrix} nc_\beta & qc_\alpha & 0 \\ qs_\beta & ps_\alpha & 0 \end{bmatrix}, \quad (22)$$

[1] Ewing, W.M., Jardetzky, W.S., and Press F., op. cit., pp 282-284.

is a solution of the fourth order system and furthermore that all combinations of separate solutions to these two second order systems are solutions of the original fourth order system. Therefore, it is possible to separate the original fourth order system into two independent second order systems. The two systems are analyzed by assuming first that $A = D = 0$ and deriving the resulting period equations for the remaining second order system. Thus

$$\varphi = B(\cosh \alpha y)e^{i(\sigma t - \xi x)}$$
$$\psi = C(\sinh \beta y)e^{i(\sigma t - \xi x)} \qquad (23)$$

which implies a motion which is symmetrical with respect to the medial plane of the plate, and consequently these vibrations are referred to as the symmetrical modes; whereas, if it is assumed that $B = C = 0$ and the other period equation is solved the functions are

$$\varphi = A(\sinh \alpha y)e^{i(\sigma t - \xi x)}$$
$$\psi = D(\cosh \beta y)e^{i(\sigma t - \xi x)} \qquad (24)$$

Since these functions define a motion which is asymmetrical about the medial plane, solutions of the corresponding period equation are referred to as the asymmetrical modes. It happens that most often the two types of modes have different phase velocities for a given frequency so that physically they are as distinct as modes of different orders, (i.e. physically, the second symmetrical mode is usually as distinguishable from the second asymmetrical mode as it is from the third symmetrical or asymmetrical mode.)

The period equation for the symmetrical case is accordingly

$$\begin{vmatrix} nc_\beta & qc_\alpha \\ qs_\beta & ps_\alpha \end{vmatrix} = 0$$

or, $npc_\beta\, s_\alpha = q^2\, c_\alpha\, s_\beta$

$$\frac{c_\beta \cdot s_\alpha}{c_\alpha\, s_\beta} = \frac{q^2}{np}$$

and substituting the original expressions back into the period equation we obtain[1]

$$\frac{\tanh \alpha\, d/2}{\tanh \beta\, d/2} = \frac{(\xi^2 + \beta^2)^2}{4\xi^2\, \alpha\beta} \qquad (25)$$

---

[1] Lamb, H., op. cit., p 116

This equation can be put into a form much more convenient for engineering work[1] by substituting the definitions of α and β. The period equation for the symmetrical case is then equal to

$$\frac{\tanh \pi fd \sqrt{\frac{V_L^2 - V^2}{V^2 V_L^2}}}{\tanh \pi fd \sqrt{\frac{V_S^2 - V^2}{V_S^2 V^2}}} = \frac{\left[2 - \frac{V^2}{V_S^2}\right]^2}{4\sqrt{1 - \frac{V^2}{V_L^2}}\sqrt{1 - \frac{V^2}{V_S^2}}} \tag{26}$$

Similarly, the period equation for the asymmetrical case is:[2]

$$\frac{\tanh \alpha d/2}{\tanh \beta d/2} = \frac{4\xi^2 \alpha\beta}{(\xi^2 + \beta^2)^2} \tag{27}$$

or:[1]

$$\frac{\tanh \pi fd \sqrt{\frac{V_L^2 - V^2}{V_L^2 V^2}}}{\tanh \pi fd \sqrt{\frac{V_S^2 - V^2}{V_S^2 V^2}}} = \frac{4\sqrt{1 - \frac{V^2}{V_L^2}}\sqrt{1 - \frac{V^2}{V_S^2}}}{\left[2 - \frac{V^2}{V_S^2}\right]^2} \tag{28}$$

2.1.6 Computation of the Period Equations

The period equations must be solved by assuming some value of V and then computing the corresponding values of frequency-thickness product by some iteration procedure. Using an IBM 709 digital computer, Worlton has found roots of these period equations for zirconium, stainless steel, uranium, brass, and aluminum, and has plotted the results as curves showing phase velocity as a function of frequency-thickness product.[3] Firestone and Ling previously made calculations for aluminum and in addition gave curves of the group velocity.[4] Normalization of the period equation permits the computation of a single set of curves for each of several values of the ratio of longitudinal velocity to shear velocity; so that the desired solution to the period equation can readily be found from these curves regardless of the material in which Lamb waves are being propagated.

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1 Worlton, D. C., op. cit., pp 9-10

2 Lamb, H., op. cit., p 122

3 Worlton, D. C., op. cit., pp 31-32, pp 51-58

4 Firestone, F. A., and Ling, D. S., Report on the Propagation of Waves in Plates - Lamb and Rayleigh Waves, Sperry Products, Inc., Danbury, Connecticut, 1945.

The period equation is normalized as follows:

Let $V_n = V/V_S$ $P = V_L / V_S$ and $(fd)_n = fd/V_S$

then for the symmetrical case:

$$\frac{\tanh \pi (fd)_n \sqrt{\dfrac{P^2 - V_n^2}{V_n^2 \; P^2}}}{\tanh \pi (fd)_n \sqrt{\dfrac{1 - V_n^2}{V_n^2}}} = \frac{\left[2 - V_n^2\right]^2}{4 \sqrt{1 - \dfrac{V_n^2}{P^2}} \sqrt{1 - V_n^2}} \tag{29}$$

and for the asymmetrical case:

$$\frac{\tanh \pi (fd)_n \sqrt{\dfrac{P^2 - V_n^2}{V_n^2 \; P^2}}}{\tanh \pi (fd)_n \sqrt{\dfrac{1 - V_n^2}{V_n^2}}} = \frac{4 \sqrt{1 - \dfrac{V_n^2}{P^2}} \sqrt{1 - V_n^2}}{\left[2 - V_n^2\right]^2} \tag{30}$$

Note that if $V > V_S$, one or both of the hyperbolic tangents on the left sides of equations 29 and 30 become circular tangents which makes the frequency-thickness product a multivalued funtion of phase velocity.

A semiautomatic program has been developed by Mr. C. L. Foreman of the University of Denver, for the Datatron 205 digital computer for computing the normalized curves of phase velocity as a function of frequency-thickness product.

Only a pilot computational effort was carried out under the present contract, as the computation of a complete set of normalized curves will require the expenditure of several thousands of dollars. The objectives of the pilot computational program were to learn more about computing the curves on the Datatron 205 digital computer and to estimate the cost of computing a complete set of curves which would include a wide enough range of ratios of longitudinal to shear velocity so that the set of curves could be used for any material that is apt to be encountered.

The symmetrical modes for the case of a ratio of longitudinal velocity to shear velocity of 1.8 were computed.

The results of this pilot computational effort are plotted as Figures 14 and 15 (Group velocity was computed using equation 39). The cost of computing a set of curves of symmetrical and asymmetrical modes giving both phase and group velocity for ratios of longitudinal velocity to shear velocity ranging in increments of .05 from 1.45 to 3.65

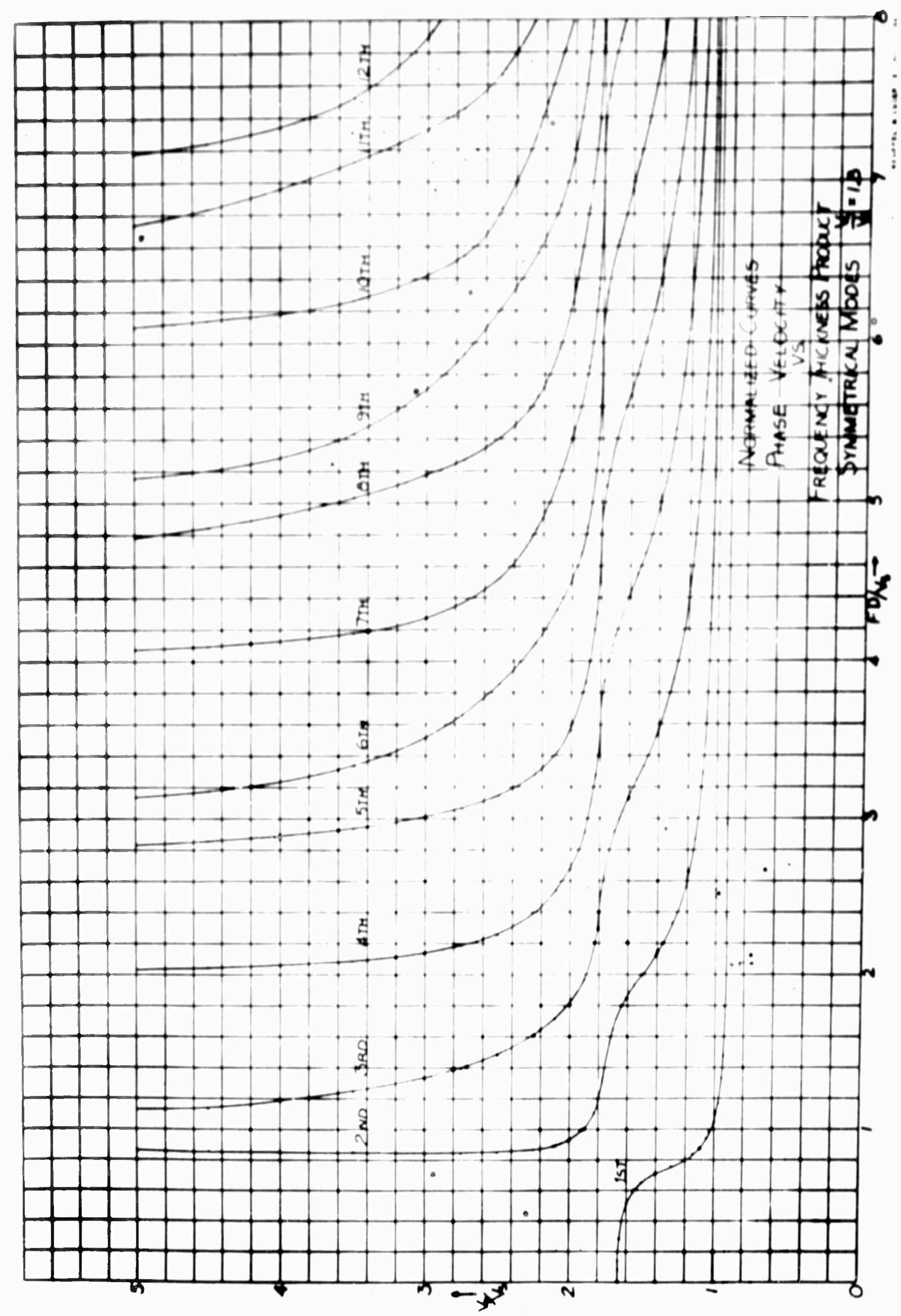
NORMALIZED CURVES
PHASE VELOCITY
VS
FREQUENCY THICKNESS PRODUCT
SYMMETRICAL MODES

FIGURE 14

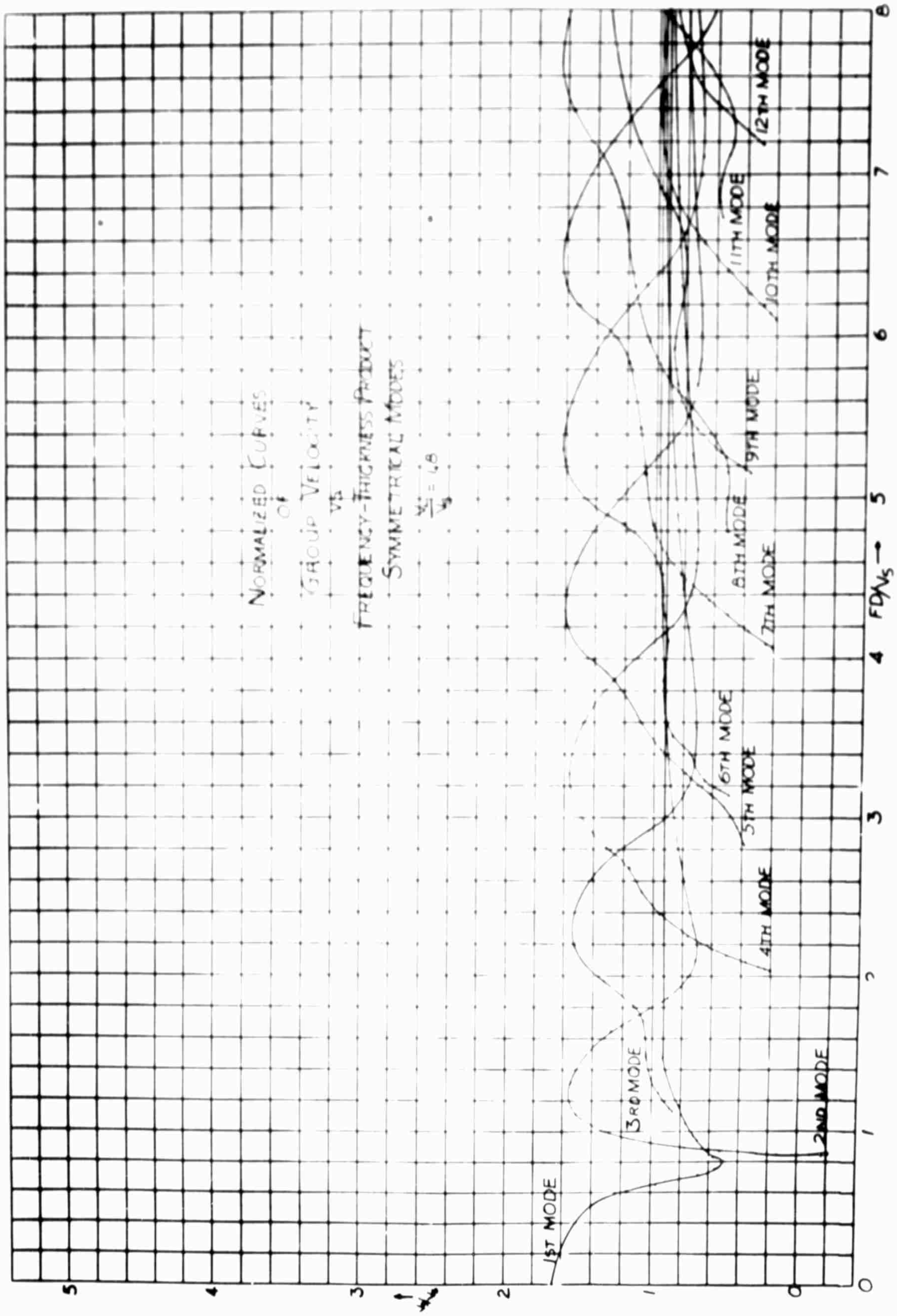
NORMALIZED CURVES
OF
GROUP VELOCITY
VS
FREQUENCY-THICKNESS PRODUCT
SYMMETRICAL MODES
1ST MODE
2ND MODE
3RD MODE
4TH MODE
5TH MODE
6TH MODE
7TH MODE
8TH MODE
9TH MODE
10TH MODE
11TH MODE
12TH MODE
0
1
2
3
4
5
6
7
8

FIGURE 15

(corresponding to a range of Poisson ratios of .0496 to 0.459) is estimated at about $25,000. This figure could be reduced to about $8,000 by using increments of .10 instead of .05 and taking the range of ratios of longitudinal to shear velocity from 1.50 to 2.60.

2.1.7 The Analogy Between Guided Sonic Waves and Guided Electromagnetic Waves

The case of waves of ultrasound being guided by a thin elastic plate is, in general, qualitatively analogous to the case of electromagnetic waves being guided by a rectangular waveguide. The reason that the analogy is only qualitative is closely related to the fact that in an extended elastic solid both longitudinal and transverse waves are propagated; whereas in an electromagnetic waveguide only transverse waves are propagated. In an extended solid, the dilational and rotational motions are propagated independently of each other. However,[1] in general, in the neighborhood of a free boundary, there is coupling between the two types of motion which is imposed by the boundary.

In particular, there is an analogy between transverse elastic waves which are polarized parallel to the free faces of the plate and electromagnetic waves in a rectangular waveguide which is quantitatively exact.[2] These transverse waves are always totally reflected as transverse waves with no coupling of energy into longitudinal waves. Elastic waves of this type are called Love waves by seismologists and are outside of the scope of this study.

Microwave engineers are concerned with minimizing the reflection coefficient at the junction between two sizes of waveguide. The problem of computing the reflection coefficient of an electromagnetic wave at such a junction is, in general, extremely complex. Consequently, minimizing the reflection coefficient is usually more of an art than a science.[3]

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[1]Mindlin, R. D., An Introduction to the Mathematical Theory of Vibrations of Elastic Plates, U.S. Army Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey, (1955), Signal Corps Contract DA-36-039 Sc 56772, p 2.21.

[2]Ewing, W.M., Jardetzky, W.S., and Press, F., Elastic Waves in Layered Media, McGraw-Hill, New York, 1957, pp 294-295.

[3]Slater, J.C., Microwave Transmission, McGraw-Hill, New York, 1942, pp 168-173.

The corresponding problem in ultrasonic inspection is to maximize the reflection coefficient by choosing a frequency and mode of propagation so that as minute a discontinuity as possible will cause a significant amount of the energy to be reflected.

2.2 Excitation of the Vibrations of Infinite Plates

2.2.1 Qualitative Description of the Spectrum of the Radio-Frequency Pulse.

Typical pulse spectra are shown in Figure 16. The shortest of these pulses is only one cycle in duration which is shorter than any of the pulses used in the experimental work to be described; however, this does not alter the general appearance of the pulse spectrum.

As is shown in the figure, when the pulse is made twice as long its spectrum covers one-half the frequency range, and when the pulse is made five times as long its spectrum covers one-fifth the frequency range. This principle of reciprocal spreading is analogous to the Heisenberg uncertainty principle of quantum mechanics.[1]

2.2.2 Fourier Integral Representation

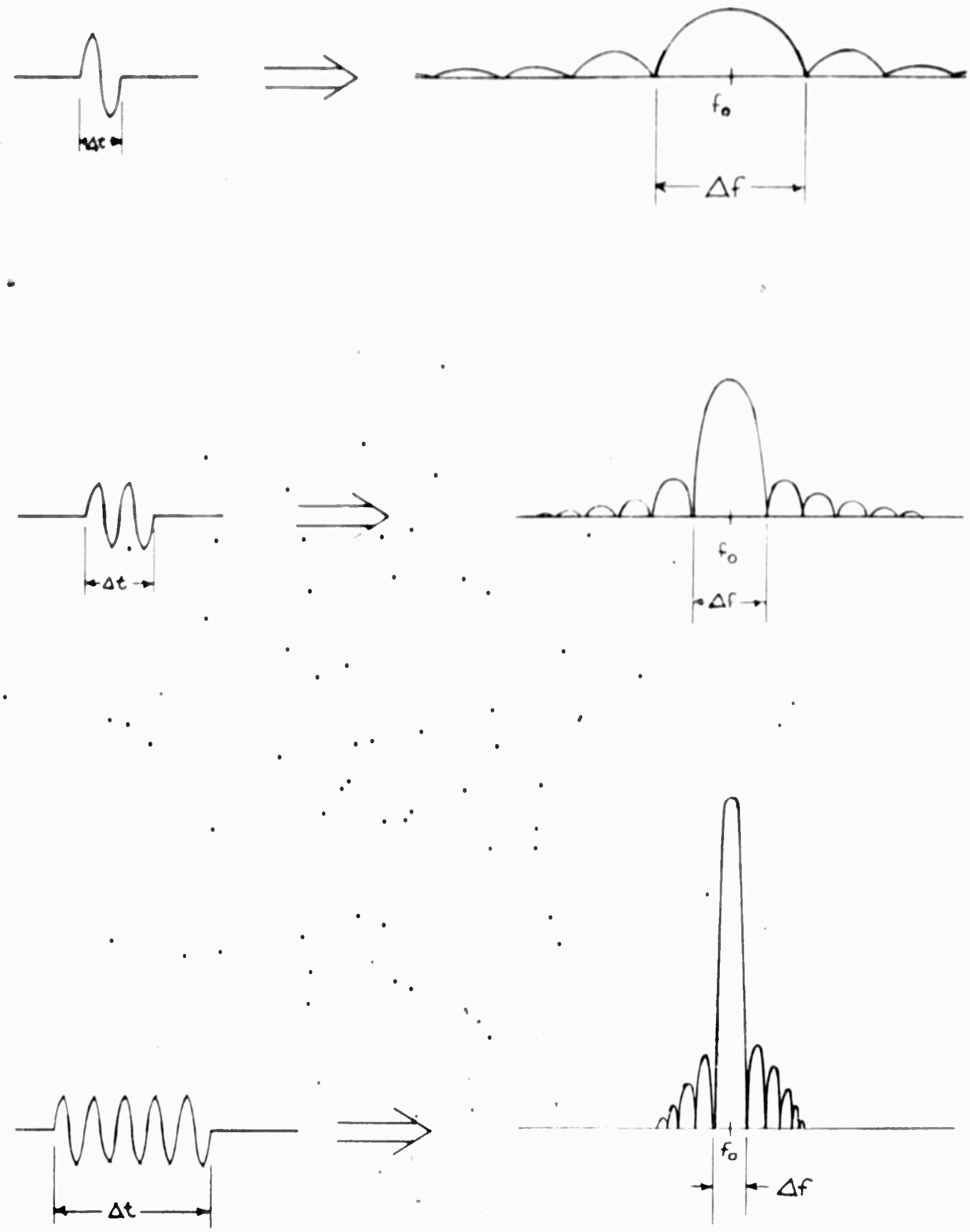
In mathematical terms, any nonperiodic function such as the pulses shown above can be expressed as a Fourier integral. In the laboratory, the waveforms used are actually periodic; however, if the pulse repetition rate is sufficiently low so that the energy of a given pulse can be assumed to have disappeared before the arrival of the following pulse, the best way to analyze the system is in terms of a single pulse.

The Fourier integral representation of a nonperiodic function, F(t) is[2]:

$$F_{(t)} = \int_0^\infty S(\sigma) \cos\left[\ \sigma t + \Phi_{(\sigma)}\ \right] d\sigma \tag{31}$$

[1]Goldman, S., Frequency Analysis, Modulation and Noise, McGraw-Hill, New York, 1948, pp 134-135.

[2]Goldman, S., op. cit., pp 53-56



PULSE DURATION × BANDWIDTH ≈ 1

FIGURE 16

$S_{(\sigma)}$ with phase $\Phi_{(\sigma)}$ is the spectrum of the function $F_{(t)}$.

$S_{(\sigma)}$ and $\Phi_{(\sigma)}$ are given by the expressions:

$$S_{(\sigma)} = \sqrt{a^2_{(\sigma)} + b^2_{(\sigma)}} \; ; \; \Phi_{(\sigma)} = \arctan \frac{-b_{(\sigma)}}{a_{(\sigma)}} \tag{32}$$

where $a_{(\sigma)} = \frac{1}{\pi} \int_{-\infty}^{+\infty} F_{(t)} \cos \sigma t \, dt$

$$b_{(\sigma)} = \frac{1}{\pi} \int_{-\infty}^{+\infty} F_{(t)} \sin \sigma t \, dt$$

Considering the case of a gated sine wave (Figure 17).

$$F_{(t)} = \begin{cases} 0 , & \text{when } t < 0 \\ \sin \sigma_O t & \text{when } 0 < t \leq t_1 \\ 0 , & \text{when } t > t_1 \end{cases}$$

$$a_{(\sigma)} = \frac{1}{\pi} \int_0^{t_1} (\sin \sigma_O t)(\cos \sigma t) \, dt$$

$$a_{(\sigma)} = \frac{1}{\pi} \left[\frac{1 - \cos (\sigma_O - \sigma) t_1}{2 (\sigma_O - \sigma)} + \frac{1 - \cos (\sigma_O + \sigma) t_1}{2 (\sigma_O + \sigma)} \right] \tag{33}$$

$$b_{(\sigma)} = \frac{1}{\pi} \int_0^{t_1} (\sin \sigma_O t)(\sin \sigma t) \, dt$$

$$b_{(\sigma)} = \frac{1}{\pi} \left[\frac{\sin (\sigma_O - \sigma) t_1}{2 (\sigma_O - \sigma)} - \frac{\sin (\sigma_O + \sigma) t_1}{2 (\sigma_O + \sigma)} \right] \tag{34}$$

Part of the spectrum of a 1 microsecond pulse of 3.3 mc. (Figure 17) was computed using these expressions. The result of this computation is shown in Figure 18. The computation was not carried any further because of the amount of labor involved. It is hoped that ultimately the digital computer can be programmed so that these formulas can be evaluated for a range of pulse duration and for several different carrier frequencies.

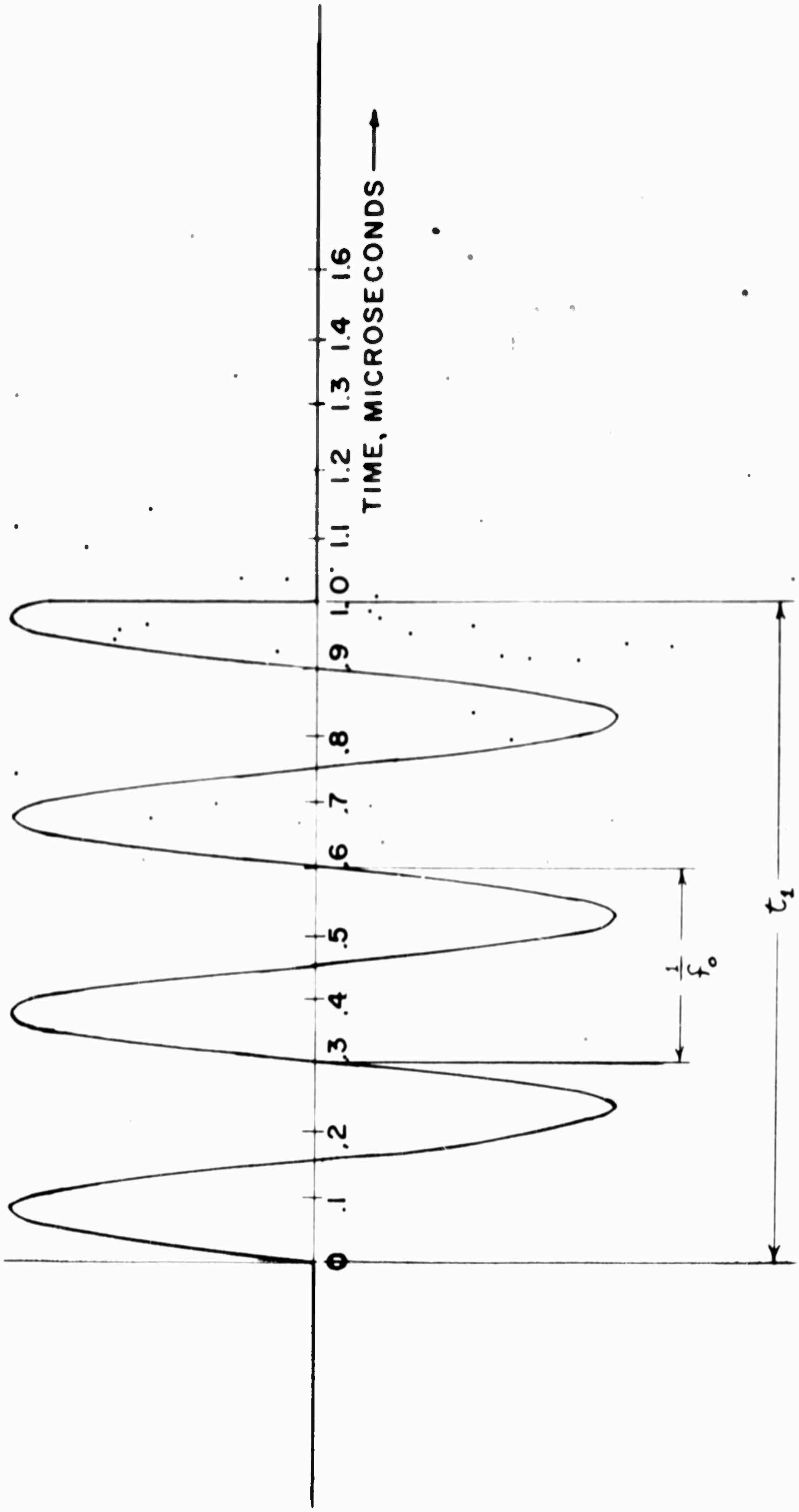
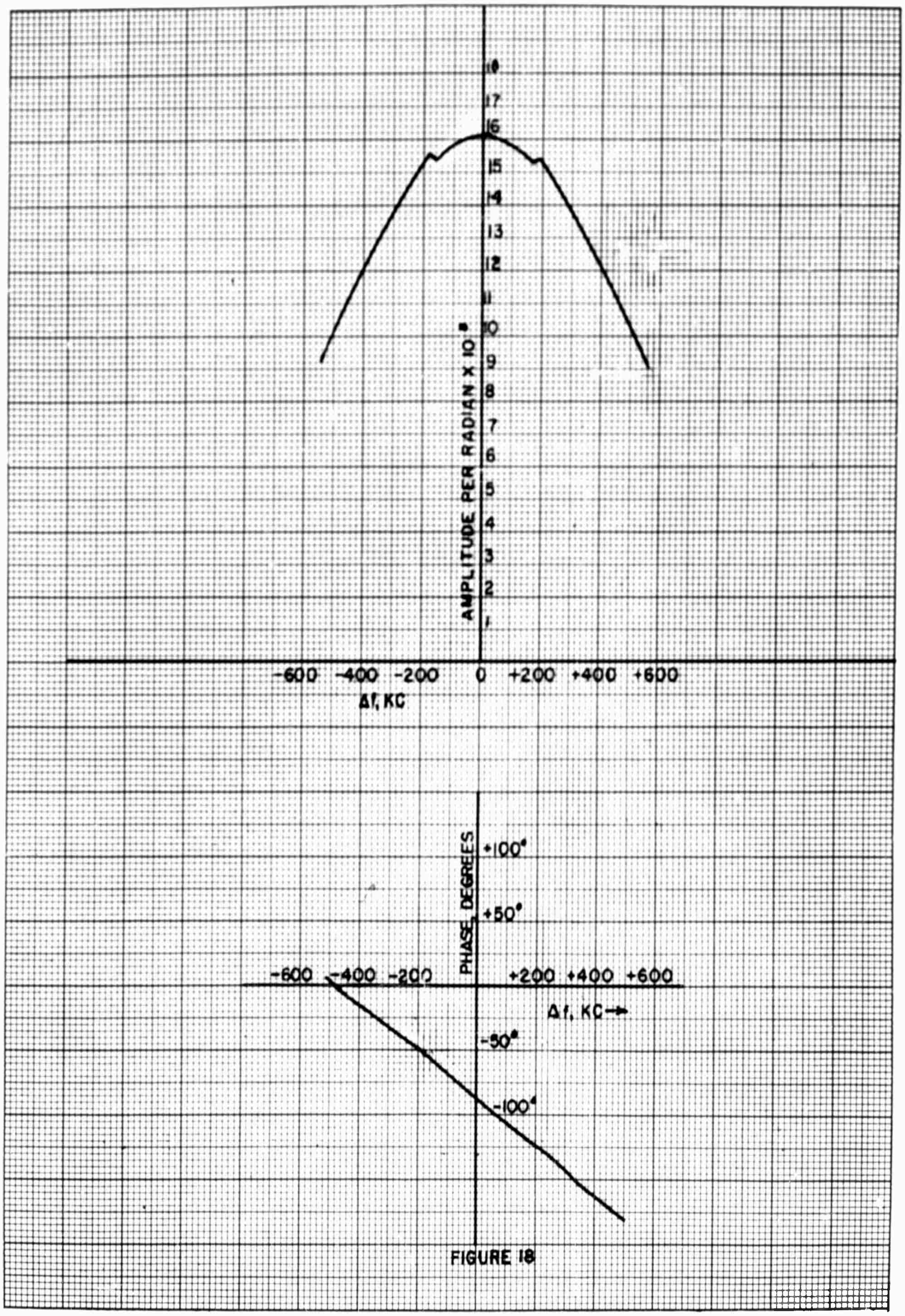


FIGURE 17



18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
AMPLITUDE PER RADIAN X 10⁸
-600 -400 -200 0 +200 +400 +600
Δf, KC
PHASE DEGREES
+100°
+50°
-600 -400 -200 +200 +400 +600
Δf, KC→
-50°
-100°

FIGURE 18

2.2.3 Principle of Stationary Phase.[1]

Given an integral of the type

$$F_{(t)} = \int_0^{\infty} S_{(\sigma)} \cos [\sigma t + \Phi_{(\sigma)}] \, d\sigma$$

where $S_{(\sigma)}$ varies much more slowly with respect to σ than $\cos [\sigma t + \Phi_{(\sigma)}]$ does. The principle of stationary phase is that under these circumstances there will tend to be calcellation of the integral over those ranges of integration where $\cos [\sigma t + \Phi_{(\sigma)}]$ is positive by the integral over those ranges of integration where $\cos [\sigma t + \Phi_{(\sigma)}]$ is negative except where

$$\frac{d}{d\sigma} [\sigma t + \Phi_{(\sigma)}] = 0$$

Points such as this where the argument of the cosine does not vary with respect to the variable of integration are called points of stationary phase. The principle of stationary phase can be applied to indicate the approximate location in time of a signal, $F_{(t)}$, by carrying out the differentiation indicated above and solving for t; so that

$$t = -\frac{d\Phi}{d\sigma} \tag{35}$$

As an example, consider the 3.3 mc. pulse of one microsecond duration considered above. From Figure 18, it may be seen that

$$-\frac{d\Phi}{d\sigma} = \frac{+\pi/2 \text{ radians}}{500{,}000 \times 2\pi \text{ radians/sec.}} = 0.5 \text{ microseconds}$$

which is indeed the approximate location of the pulse, since it was originally assumed to extend from $t = 0$ to $t = 1$ microsecond.

2.2.4 Phase Velocity and Group Velocity

Phase velocity is defined as the velocity with which an individual wave crest travels. Group velocity is defined as the velocity with which a group of waves travels. In Figure 19 is shown an r-f pulse of four cycles at some time, t_1, and at some later time, t_2. In the interval $t_2 - t_1$ the pulse has traveled a distance $(t_2 - t_1)V_g$ where V_g is group

[1] Goldman, S., op. cit., pp 111-112.

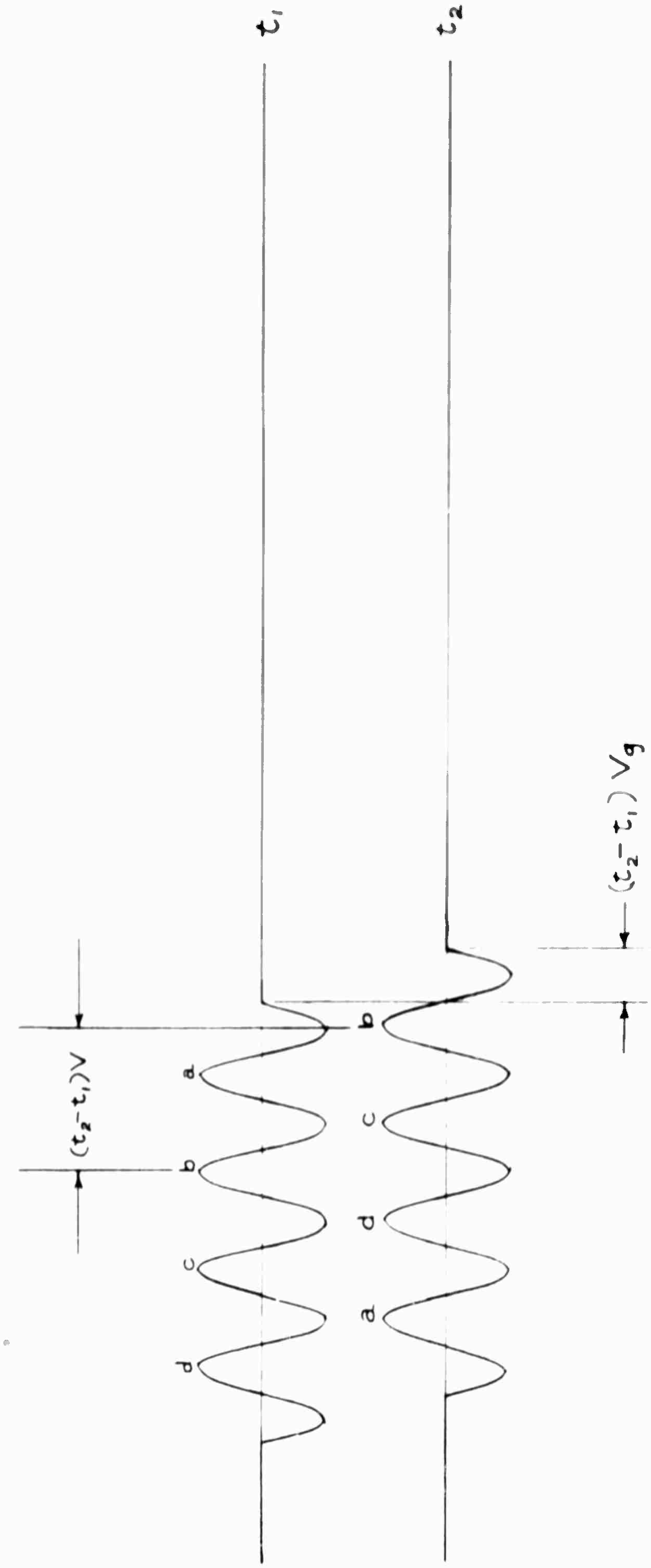


FIGURE 19

velocity. In this same interval of time the wave crest b, for example, has traveled the distance $(t_2 - t_1)V$ where V is phase velocity. During this interval, the wave crest designated "a" has moved to the front of the group, disappeared, and reappeared at the rear to start moving towards the front of the group again. The case shown is the one in which the group velocity is less than the phase velocity. When the group velocity is greater than the phase velocity, the individual wave crests move toward the rear of the group, disappear, and reappear at the front.

The principle of stationary phase discussed above can be used to derive the formula relating phase and group velocity. The expression for a traveling plane wave of one frequency traveling parallel to the x axis is

$$A \cos \sigma \left(t - \frac{x}{V}\right) \tag{36}$$

However, the r-f pulses discussed above are made up of spectra of frequencies; furthermore, V is a function of frequency. Writing equation 31 in a form corresponding to equation 36,

$$F_{(t)} = \int_0^{\infty} S_{(\sigma)} \cos \sigma \left(t - \frac{x}{V}\right) d\sigma,$$

and applying the principle of stationary phase to locate the pulse in time one obtains:

$$t = x \left[\frac{1}{V} - \frac{\sigma}{V^2} \cdot \frac{dV}{d\sigma}\right]$$

Since by the definition of group velocity, V_g,

$V_g = x/t$ and since, $\xi = \sigma/v$,

it is clear that $1/V_g = \dfrac{d\xi}{d\sigma}$ and that

$$V_g = \frac{d\sigma}{d\xi} \tag{37}$$

since $\sigma = V\xi$,

$$V_g = V + \xi \frac{dV}{d\xi} \qquad \text{and since } \frac{dV}{df} = \frac{dV}{d\xi}\,\frac{d\xi}{df} \quad ,$$

$$\frac{dV}{d\xi} = \frac{-\dfrac{dV}{df}}{\dfrac{\xi}{V}\left[\dfrac{dV}{df} - \dfrac{V}{f}\right]} = \frac{-V}{\xi\left[1 - \dfrac{V}{f\dfrac{dV}{df}}\right]}$$

$$\therefore \quad V_g = V\left[1 - \frac{1}{1 - \dfrac{V}{(fd)\dfrac{dV}{d(fd)}}}\right] \qquad (38)$$

To normalize this expression with respect to shear wave velocity, let $V_{gn} = V_g/V_S$ and since $V_n = V/V_S$ and $(fd)_n = fd/V_S$, equation 38 may be written

$$V_{gn} = V_n\left[1 - \frac{1}{1 - \dfrac{V_n}{(fd)_n\dfrac{dV_n}{d(fd)_n}}}\right] \qquad (39)$$

which is the equation used to compute the normalized curves of group velocity versus frequency-thickness product shown in Figure 15.

2.2.5 Trace Velocity

As used in this report, the trace is defined as the intersection of the incident wavefront with the upper surface of the plate. The velocity with which the trace travels across the plate is defined as the trace velocity. The purpose of this definition is to differentiate between this velocity, which is a function of only the incident angle and the medium in which the incident wavefront exists, and the phase velocity, which is a property of the Lamb wave itself and is a function of the frequency, and the material and thickness of the plate.

The following derivation is intended to convey an intuitive concept. It is not rigorous but it is thought that a rigorous derivation may evolve from it.

In Figure 20 is shown an enlarged view of a wedge, a transducer, and the underlying section of plate. The intersection of the beam with the surface of the plate has a length shown as the interval from $x = 0$ to $x = x_1$. The effect of the incident wave front in inducing a Lamb wave in the plate is represented by an average force, F, acting normal

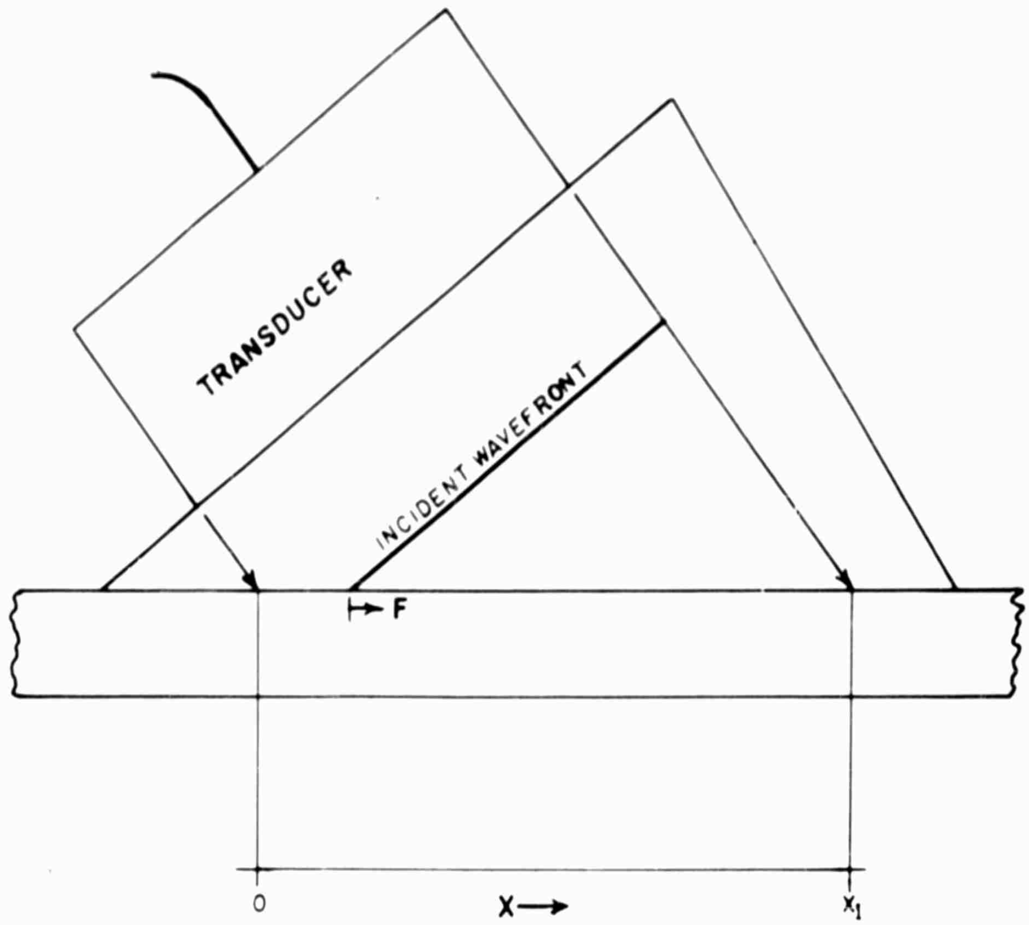


FIGURE 20

to the trace and moving with it across the plate. When the trace and phase velocity are exactly equal, the amplitude of the Lamb wave induced in the plate will be proportional to the product $Fx_1 = Eo$. When the phase and trace velocity are not exactly equal, the force, F, and the Lamb wave gradually get out of phase as they travel across the plate. The phase difference between them is given by

$$\theta = \left(\frac{2\pi f}{V_T} - \frac{2\pi f}{V}\right) x \qquad \text{where}$$

V_T = Trace velocity and V = phase velocity.

If V_T is slightly less than V or $V_T = V - \Delta V$

$$\theta = \frac{2\pi f}{V} \cdot \left(\frac{\Delta V}{V - \Delta V}\right) x \approx \frac{2\pi f}{V} \left(\frac{\Delta V}{V}\right) x$$

The effectiveness of the incident wave in inducing a Lamb wave in the plate is now given by

$$E = \int_0^{x_1} F \cos \theta \, dx$$

$$\text{let } m = \left(\frac{2\pi f}{V}\right) \frac{\Delta V}{V}$$

$$\text{then } E = \int_0^{x_1} F \cos (mx) \, dx$$

$$E = \frac{F \sin (mx_1)}{m}$$

$$\text{so that } E/E_o = \frac{\sin (mx_1)}{mx_1}$$

The assumption that the transducer is rectangular rather than round is implicit in the above derivation. The efficiency of a round transducer such as was actually used in the experimental work will fall off less steeply with respect to the difference between phase and trace velocity.

In practice, the frequency is usually adjusted for maximum efficiency; however, the frequency can be varied somewhat about the point of maximum efficiency before there is a large change in efficiency.

EXPERIMENTAL WORK

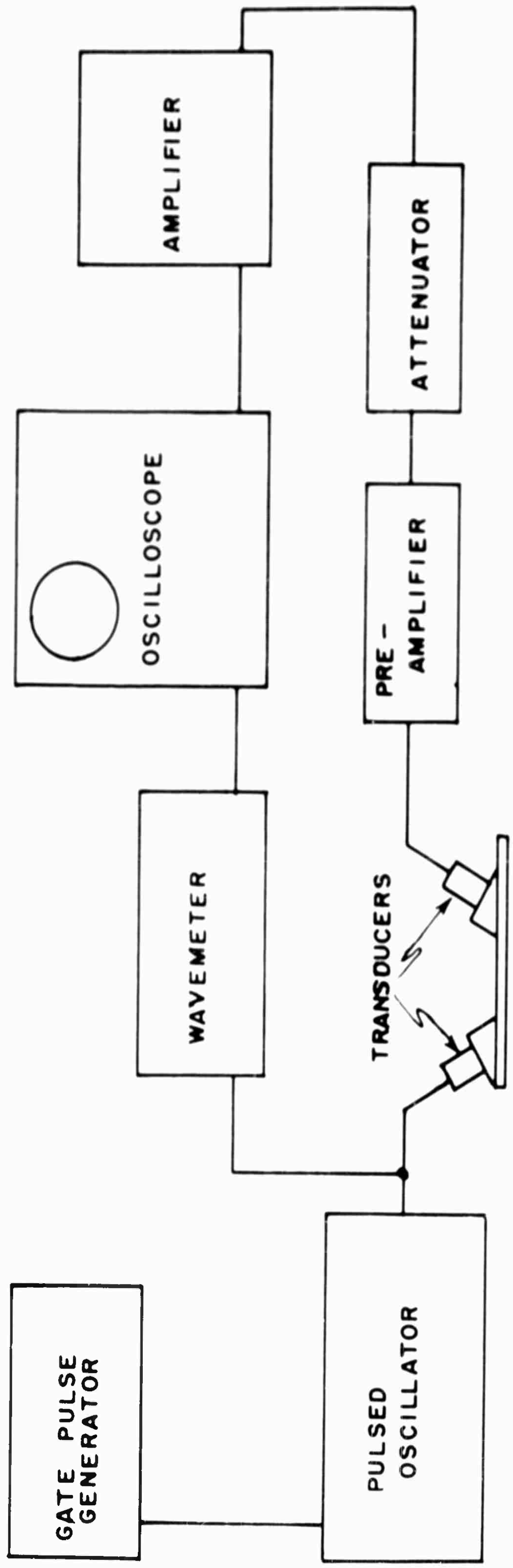
3.1 Introduction

Some of the experimental work performed during the earlier part of this project was intentionally chosen to overlap the work of prior investigators. By this means it was possible for the authors to acquire confidence both in their qualitative understanding of Lamb Waves and in their apparatus, which differed somewhat from that of previous investigators. The most important respect in which their apparatus differed was in the use of lucite wedges for obtaining the desired angle of incidence. Firestone and Ling used Y-cut quartz crystals and Worlton used immersion techniques.

3.2 Apparatus and Materials

The functional block diagram of the electronic apparatus is shown in Figure 21. The requirements of the various components are as follows: The gate pulse generator must supply a single-polarity pulse of variable duration and pulse repetition frequency and of sufficient amplitude to fully enable the pulsed oscillator. The pulsed oscillator should begin oscillating after it receives the gate pulse and continue to oscillate throughout the duration of the gate pulse at a frequency determined by its internal tank circuit. The primary requirements for the pulsed oscillator are that it produce a large amount of r-f voltage with as purely a sinusoidal waveform as possible, and that it be capable of being gated to produce pulses of variable duration. The transducers should have as high a power conversion efficiency as possible which implies a high electromechanical coupling factor. The pre-amplifier should have as low a noise factor as possible. The attenuator is used for making measurements of insertion loss and consequently should be a precision attenuator. The amplifier should provide as much gain as can be used without overdriving the oscilloscope; noise factor should not be critical since it is assumed that the pre-amplifier has already amplified the signal to a level considerably above the noise level of the amplifier. The oscilloscope must have the proper bandwidth and a selection of sweep speeds. The wave meter is for providing a permanent frequency reference so that any change in oscillator calibration can be determined. The required bandwidth to the 3 db points of the overall system about the carrier frequency is given by the empirical formula:

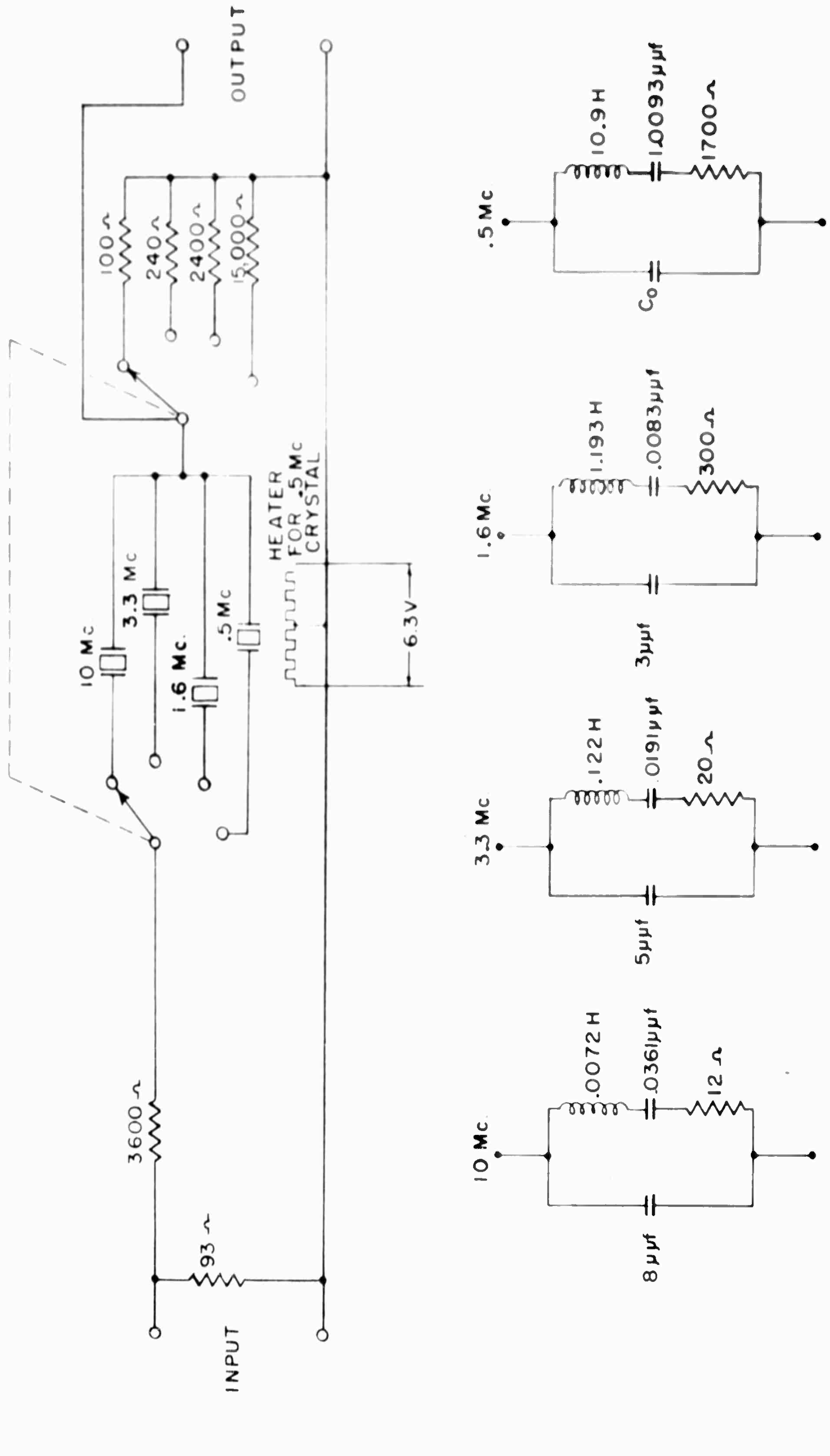
$$\text{Bandwidth} = \frac{75}{\text{Rise Time}}$$



FUNCTIONAL BLOCK DIAGRAM OF ELECTRONIC APPARATUS

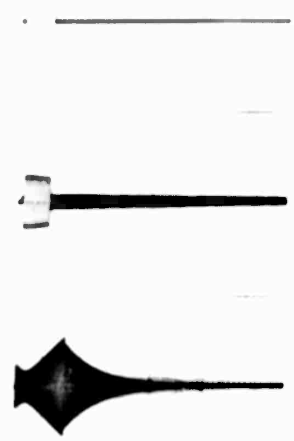
FIGURE 21

The components actually used were as follows The gate pulse generator used is an Electropulse Model 4120A variable pulse generator. The pulse duration of this pulse generator is variable from 0.3 microseconds to 100 microseconds, and the pulse repetition frequency is variable from 33.3 cps to 330 kc. The pulsed oscillator used is an Arenberg PG-650-C which has a rated r-f output of 0-300 volts for a rated pulse length of 2.5 to 10 microseconds. When the pulse length is made 100 microseconds by the use of an external gate pulse, the r-f output is somewhat less than this. The PG-650-C has an internally generated gate pulse which was also used at times on this project; however, it was often desirable to use a pulse longer than 10 microseconds to narrow the frequency spectrum because of the fact that Lamb waves are dispersive The transducers used are Branson type Z-.5 inches in diameter. One pair of transducers was obtained for each of the frequencies of 0.5 mc., 1.6 mc., 3.3 mc and 10 mc. The pre-amplifier used consists of two Hewlett-Packard type 460A wide band amplifiers in cascade. These amplifiers each have 20 db gain and have a rated noise factor of less than 10 db. The bandwidth is ten times as great as is required for this particular application, but they were already on hand when the project commenced. The only disadvantage that these amplifiers possess for this type of work is that they are designed to be used with a special 200 ohm coaxial cable which is physically large and might be somewhat inconvenient in a practical testing situation; although, it was satisfactory in the laboratory The attenuator used is an Arenberg type No. ATT 693 precision attenuator, which has a range of 0-122 db in 1 db steps at an impedance level of 93 ohms with a rated maximum absolute error of 1 db. The amplifier used is a Tektronix type No. 121 wide-band oscilloscope preamplifier with a voltage gain of 40 db and a pass band of 5 cps to 12 mc. The oscilloscope used is a Tektronix type No. 545 equipped with a type No. 53/54C dual-trace preamplifier. The combination has a bandwidth of 24 mc.; the calibrated sweep range extends from 5 sec./cm to .02 μ sec per cm. The wavemeter was constructed using quartz crystals resonant at 0.5000 mc., 1.6000 mc., 3.3000 mc and 10.000 mc. A circuit diagram is shown in Figure 22. The 3600 ohm resistor is to protect the crystals from being cracked by the full output voltage of the pulsed oscillator. When the duration of the pulse is ten microseconds its frequency spectrum is so broad that very little change can be noted in the amplitude of the pulse at the output of the wave meter. However, the energy stored in the crystal during the pulse is very much a function of the carrier frequency and consequently the magnitude of ringing after the cessation of the pulse is maximum when the carrier frequency is equal to the resonant frequency of the crystal. Figure 23 shows the waveform at the output of the wave meter for pulses of 10 microseconds with carrier frequency appreciably different from the resonant frequency



WAVE METER CIRCUIT DIAGRAM

FIGURE 22



U. 10 Microsec. Div.
M. 10 Microsec. Div.
L. 100 Microsec. Div.

FIGURE 23

of the crystal (upper waveform), 10 microseconds very near resonance (middle waveform), and 170 microseconds very near resonance (lower waveform). The finished wave meter chassis may be seen in the background of Figure 24. The wave meter was used to obtain an experimental estimate of the width of the spectra of pulses of various lengths. To do this, the oscillator frequency was varied and the amplitude of the "tail" at the end of the pulse was plotted as a function of oscillator dial reading. This is shown in Figure 25; the range of dial readings of 74 to 94 shown corresponds to a variation in frequency from 3.06 mc. to 3.38 mc. These curves give a good indication of the frequency spectra even though the response of a fixed-frequency wave meter as the oscillator frequency is varied is not the same as the response of a variable-frequency wave meter as it is tuned over the spectrum of a given pulse.

Photographs of waveforms were taken on Polaroid type No. 46-L Land film using a Dumont type No. 302 oscillograph recording camera. This type of film yields positive transparencies which were filed. Ozalid copies were made for pasting in the laboratory notebook.

The transducer positioning clamp (Figure 24) was constructed at Denver Research Institute for use in other ultrasonic investigations, and proved satisfactory for the present project.

Two sizes of test plates were used. The experiments described in sections 3.3 thru 3.9 were made using plates of starrett type No. 496 precision ground stock 10" × 18" in the thicknesses 1/32", 1/8", 3/16", 1/4", 5/16", 13/32", and 1/2". This stock is supplied to a tolerance of ±.001". Experiments using test plates of this size showed that a smaller plate should work equally well, as no echoes were received from the sides of the plate. The smaller size of test plate is 2" × 10". Nine of these are obtained by cutting one of the 10" × 18" plates. Since the plate should be bounded by air on both sides, a frame was constructed for each of the two sizes of plates. These frames were made by bending a lip on each of four strips of aluminum and joining them with angle brackets which projected slightly above the plate. The frame for the large plates may be seen in Figure 24.

In order to excite Lamb waves in the plates it was desired to project an incident longitudinal wave at varying angles of incidence. Since the first asymmetrical mode may have phase velocities that range all the way from zero to Rayleigh wave velocity, it was desirable to have a medium for the incident wave with as low a longitudinal velocity as possible, since the trace of the incident wave front cannot have a velocity

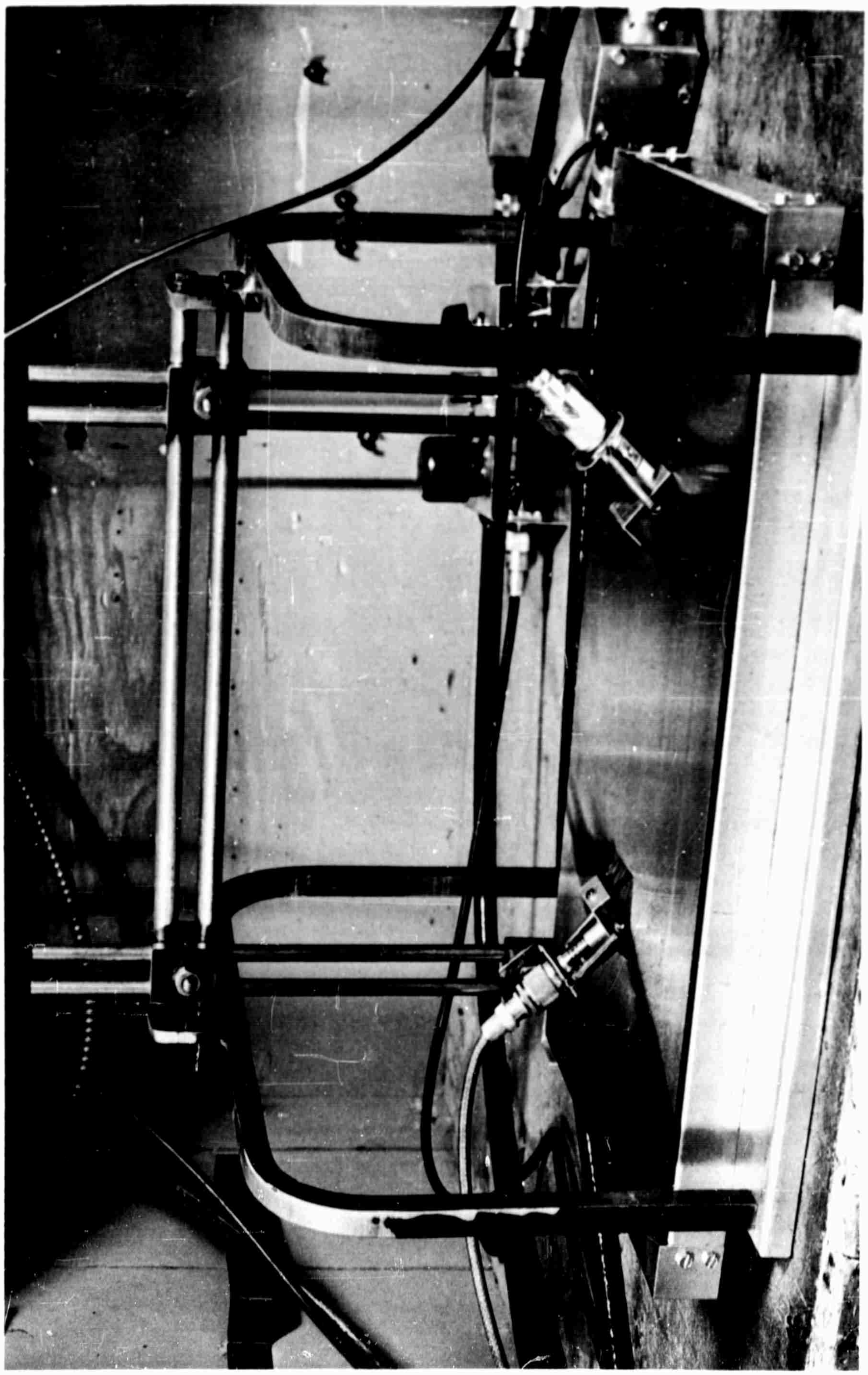
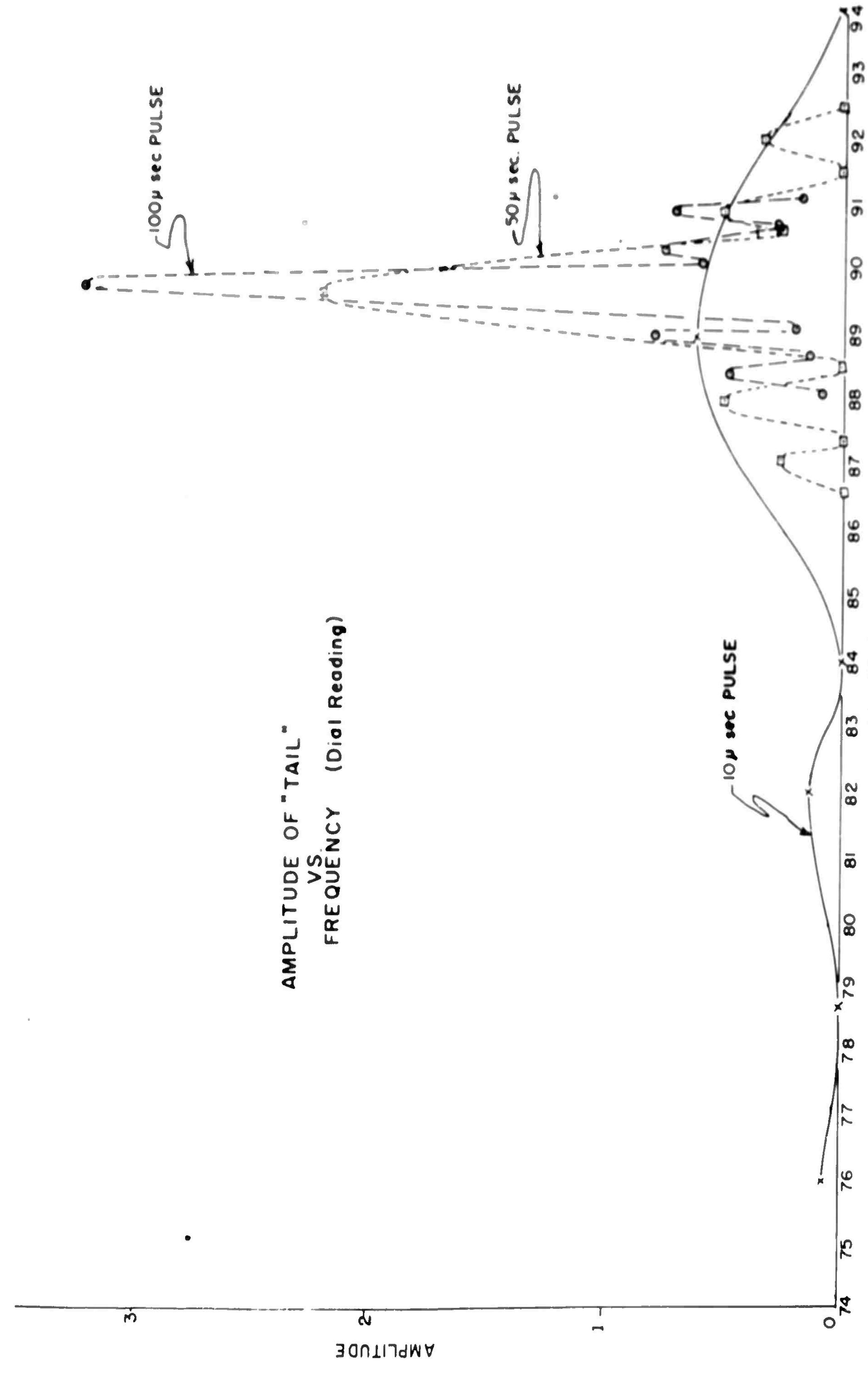


FIGURE 24



DIAL READING
FIGURE 25

any lower than the longitudinal velocity in this medium. It was also desired to use solid wedges for this purpose, since there is little mentioned in the literature about the use of wedges for the excitation of Lamb waves. Lucite was used for most of the wedges. The lucite wedges were cut from 1.25" square stock on a precision miter box. The wedge angle, as the term is used in this report, is equal to the angle of incidence of the sound beam measured from the normal. In order to keep the transducer positioned over the same part of the wedge at all times, aluminum plates were attached to the wedges with machine screws, and each plate had a hole punched in it which was of the same diameter as the transducer. In order to keep the transducer in contact with the wedge, springs were attached to the wedge with machine screws at one end and looped over projections on an aluminum fitting at the other (Figure 24). In order to obtain lower phase velocities than could be obtained with lucite, some wedges were cut from polystyrene which has a lower longitudinal velocity. Since the cost of obtaining 1.25" square polystyrene stock was prohibitive, wedges were made which were only .25" wide and which were quite satisfactory (Figure 26).

The couplant used was ordinary motor oil.

3.3 Ray Theory

Since in using longitudinal and shear waves of ultrasound in nondestructive testing it is usually possible to understand the situation in terms of ray diagrams, it was thought worthwhile to try to apply them to the problem of the transmission of ultrasound along plates. If the plate is sufficiently thick and the angle of incidence exceeds the critical angle for longitudinal waves, the situation is as pictured in Figure 27. The sound may be received only at discrete intervals along the plate and (within the plate) it will be delayed with respect to the transmitted pulse an amount equal to $2nd/V_S\cos\theta$ where n is the number of times the sound has been reflected from the bottom surface of the plate, d is the thickness of the plate, and θ is the angle that the ray makes with the normal to the surfaces of the plate. If the plate is made half as thick the only effect is to cause the sound to be reflected from the bottom twice as many times as shown in Figure 28. It is intuitively clear that in neither of these cases is the waveform or the delay time dependent upon frequency. In Figure 29 is shown the effect of making the plate very thin, so that many reflections of the two rays shown occur at the interface between the plate and the receiving wedge. It appears that the frequency will now have to be considered since the transmission of sound from the transmitting wedge to the plate will be good only when there is constructive interference.

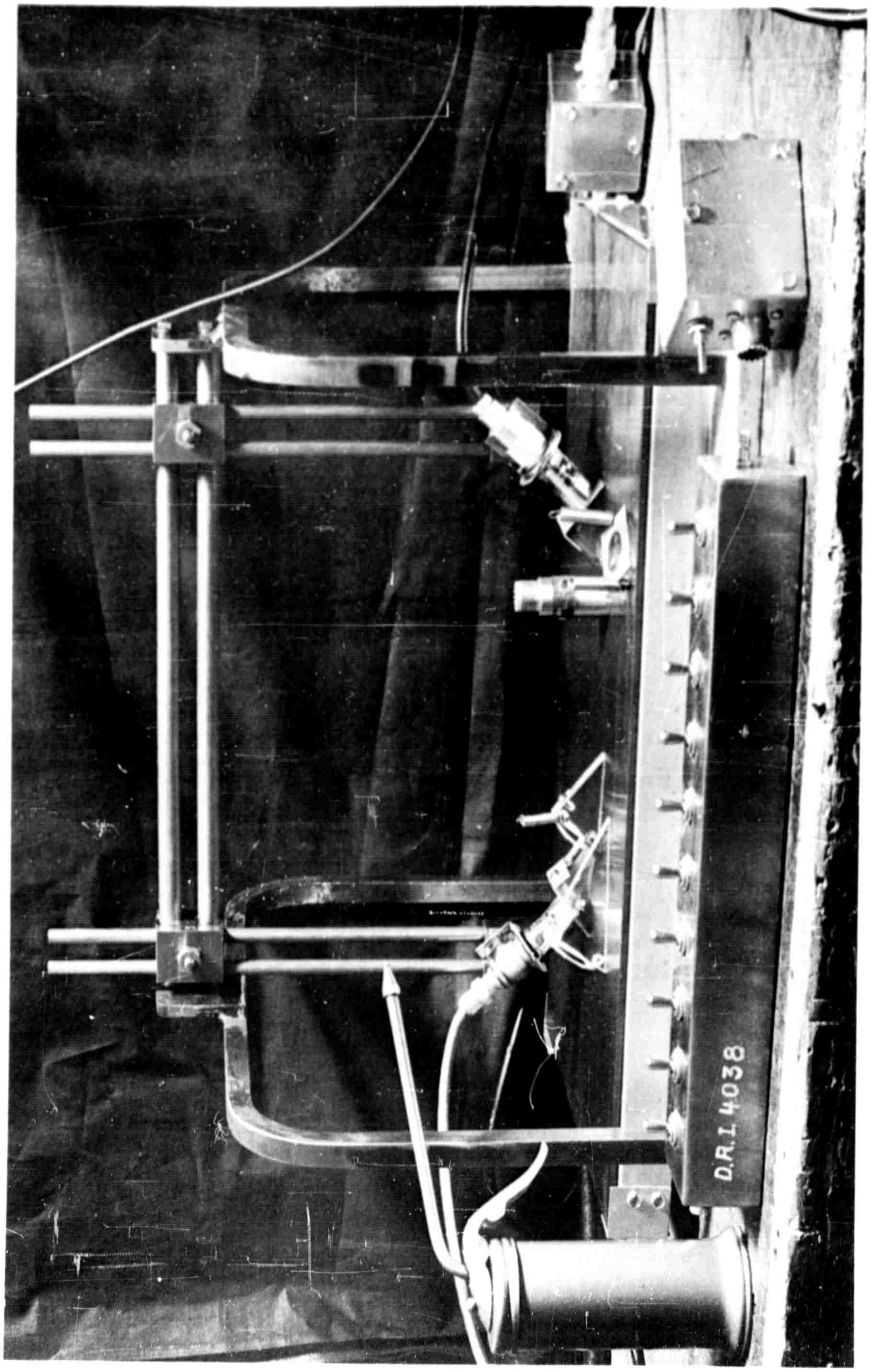


FIGURE 26

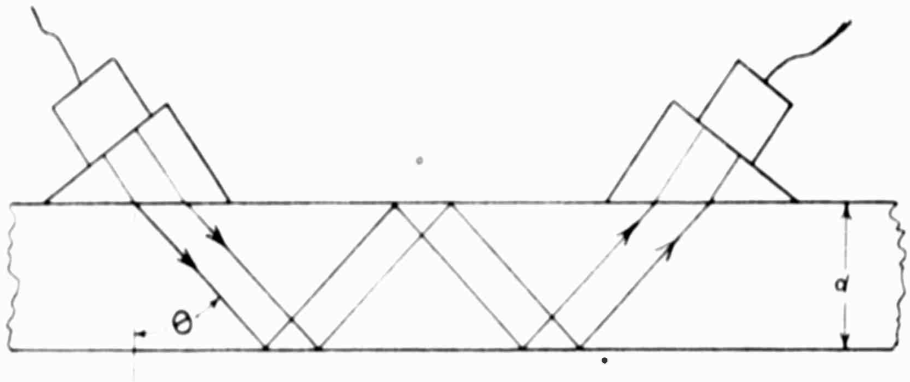


FIGURE 27

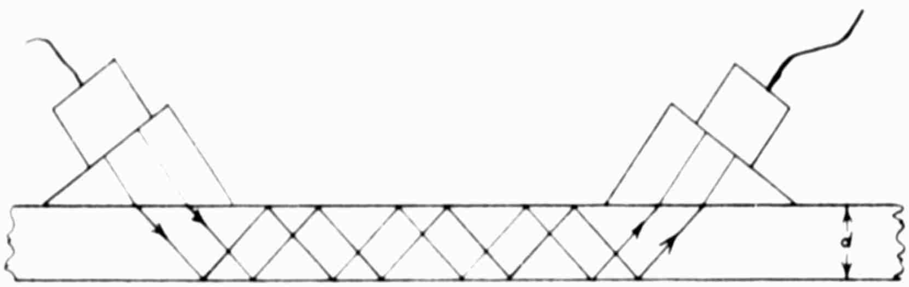


FIGURE 28

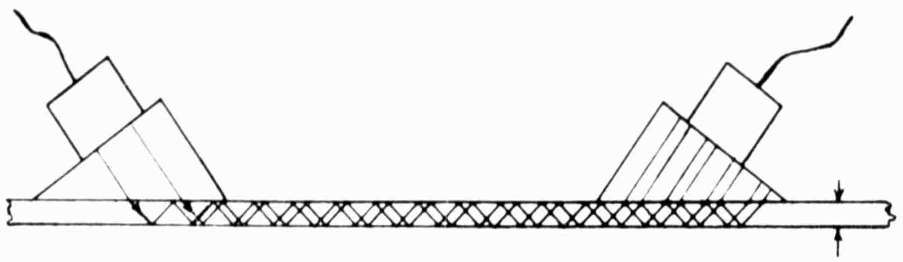


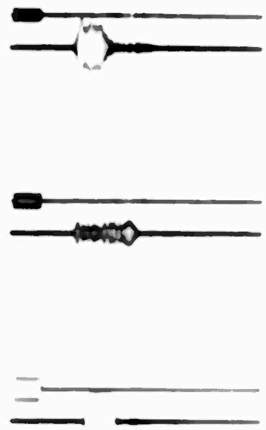
FIGURE 29

If it is assumed that the frequency has been adjusted so that optimum transmission occurs from the wedge to the plate, then by symmetry the condition for optimum transmission from the plate to the receiving wedge is also satisfied. Since the ray from the right side of the transmitting transducer now produces a ray at the left side of the receiving transducer, the above formula for delay time is no longer accurate for short distances. Furthermore, since each ray from the transmitting transducer produces several rays at the receiving transducer, some stretching of the pulse is to be expected. However, it appears that the delay time should not vary with frequency. The experiment performed to test this simple ray theory consisted of using a plate of aluminum .0502 inches thick and two lucite wedges placed as in Figure 29. The wedges were cut so that the angle of incidence in the lucite would be 25.3° which is slightly greater than the critical angle for longitudinal waves in aluminum. Figures 30 and 31 show the effect of varying the frequency while holding the distance between transducers constant. There are three exposures on each photograph. The upper waveform of each exposure is the output of the wave meter and is shown to indicate the time at which the pulse was applied to the sending transducer. The lower waveform of each exposure is the amplified pulse from the receiving transducer. In both of these photographs, the upper exposure was taken at 3.44 mc., the middle exposure was taken at an intermediate frequency and the lower exposure was taken at 2.28 mc. In Figure 30 the separation between wedges was 1.64 in., in Figure 31 the separation between wedges was 3.68 in.

It is clear from these photographs that the delay time varies with frequency. The group velocity as predicted on the basis of a multiply reflected shear wave is equal to $V_S \sin \theta$ If the numerical values are substituted in this case, the calculated group velocity is 51,200 in./sec. By measuring the delay at the two different values of transducer separation and dividing the difference in delay by the difference in separation, the group velocities at 2.28 mc. and 3.44 mc. were found to be 125,100 in./sec. and 163,000 in./sec. respectively. The ratio of measured velocity to calculated velocity is thus 2.5 at 2.28 mc. and 3.3 at 3.44 mc. which shows that the propagation of ultrasound in this plate can not be explained in terms of ray theory.

3.4 Plate Thickness

An effort was made to determine how great the thickness of a plate must be relative to a wavelength before it ceases to act as a plate. A piece of magnesium 1.22 " thick was tested to find out what to expect

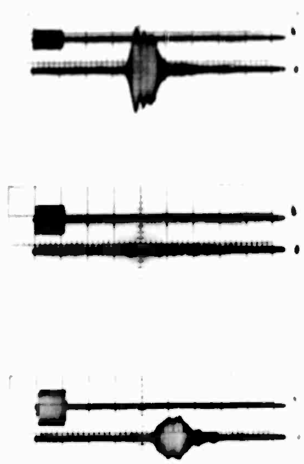


10 Microsec./Div.

10 Microsec./Div.

10 Microsec./Div.

FIGURE 30



10 Microsec./Div.

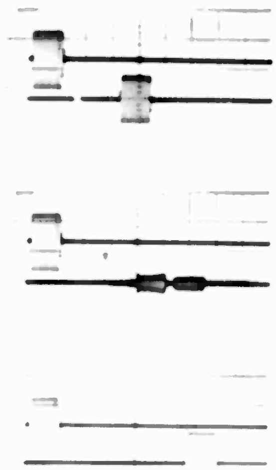
10 Microsec./Div.

10 Microsec./Div.

FIGURE 31

from a very thick plate. Using 35° wedges and a frequency of 2.25 mc., the distance and time between successive reflections were found to be 5.30 cm. and 27 microseconds respectively. These values agreed with the calculated values of 5.32 cm. and 26.8 microseconds. Figure 32 shows how the waveform changes as the distance between transducers is increased. The upper trace of each exposure is the output of the wave meter to show the time of the transmitted pulse. In the upper exposure, the distance between transducer supporting rods was 9.3 cm., in the middle exposure, the distance between transducer supporting rods was 12.1 cm. and the lower exposure, the distance between transducer supporting rods was 14.6 cm.

The 1/2" thick steel plate was investigated next and was also found to act as an extended solid bounded by two parallel planes which repeatedly reflected the sound back and forth between them. The 5/16" plate behaved in this manner as did the 1/4" plate. The distances between the transducer supporting rods for six successive points of maximum amplitude on the 1/4" plate using 35° lucite wedges were found to be 11.77 cm., 13.00 cm., 14.20 cm., 15.48 cm., 16.78 cm., and 18.12 cm. The differences between successive points are thus 1.23 cm., 1.20 cm., 1.28 cm., 1.30 cm., and 1.34 cm., which averages 1.27 cm. This agrees with the calculated value of 1.21 cm. within the limits of experimental error. Figure 33 shows the waveforms obtained in this case. The upper exposure shows the received pulse when the transducer supporting rods are separated by 13.90 cm.; the middle exposure shows the received pulse for a separation of 15.30 cm. with transmitting transducer in the same position as in the upper exposure. The lower exposure shows the received pulse with the time scale expanded. The lower waveform in each exposure is the output of the wave meter. When this same experiment was performed on the 1/8" plate, the successive reflections tended to blend into one another, and the group velocity seemed to change as the frequency was varied. The 1/32" plate definitely acted as a plate. No recognizable reflections were observed, and the group velocity changed as the frequency was varied. The 1/32" plate is approximately one wavelength thick and behaved as a plate; whereas the 1/4" plate is eight wavelengths thick and behaved as an extended solid bounded by two parallel planes. The conclusion drawn is that this apparatus produces Lamb waves when the thickness of the plate is of the order of one wavelength, and reflected shear waves when the thickness of the plate is an order of magnitude greater than one wavelength.

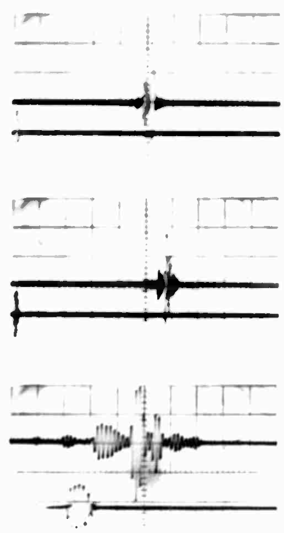


10 Microsec./Div.

10 Microsec./Div.

10 Microsec./Div.

FIGURE 32



10 Microsec./Div.

10 Microsec./Div.

2 Microsec./Div.

FIGURE 33

Figure 34 was taken using the 1/8" plate with 20° lucite wedges and a separation between transducer supporting rods of 21.6 cm. The upper waveform was taken at 3.04 mc., the middle waveform was taken at 3.14 mc., and the lower waveform was taken at 3 23 mc. The time scale for all three waveforms was 20 microseconds per division. The second pulse results from Lamb wave propagation and the group velocity clearly varies with frequency.

3.5 Beam Divergence

In the above discussion, it was assumed that there was no beam divergence. In terms of ray theory, the angle from the axis to the most divergent ray when the transducer can be considered to be a circular piston (Figure 35) is sometimes assumed to be given by the formula

$$\phi = \sin^{-1}\left(.61 \frac{\lambda}{r}\right)$$

where λ is the wavelength and r is the radius of the transducer This formula actually gives the angle to the first zero of the Fraunhofer diffraction pattern. When interpreting the situation in terms of rays, the Fresnel zone and the secondary lobes of the Fraunhofer pattern are frequently ignored However, if the above formula is used to calculate the magnitude of beam divergence, some insight is gained into the reason for obtaining the type of waveforms shown in Figures 36 and 37 In Figures 36 and 37, the frequency was 2 41 mc., the wedge angles were 40°, and the plate thickness was 0.25 in The lower exposure in each case shows the output of the wave meter in addition to the received pulse The middle exposure shows the received waveform when the pulse width is 15 microseconds; the upper exposure shows the received waveform when the pulse width is 2 5 microseconds. In Figure 36, the separation between transducer supports was 25 0 cm In Figure 37, the separation between transducer supports was 10.9 cm. As the transducers were moved further apart, a greater number of pulses were contained in the received waveform. At first it was thought that these different pulses represented different modes of Lamb waves traveling with different group velocities so that the different modes lagged behind one another more and more as they traveled along the plate. However, if this were true it should be possible to see a given pulse gradually separate into two different pulses as the transducers were moved further apart. This effect was not observed and it was decided that this phenomenon could be explained in terms of beam divergence. If ϕ is computed, using the above formula at 2 41 mc. for lucite and a .5 in.

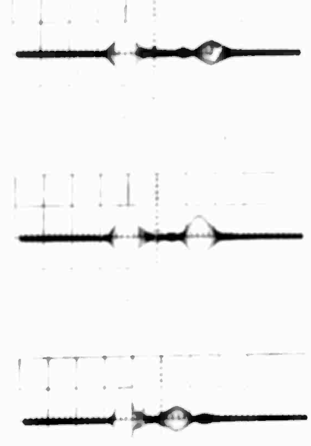


FIGURE 34

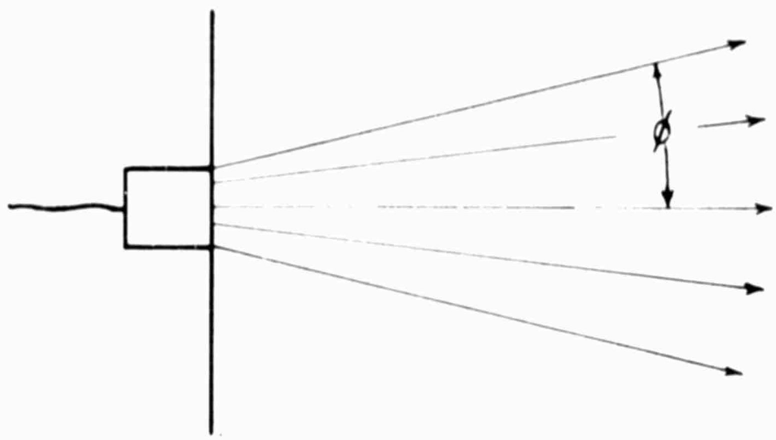


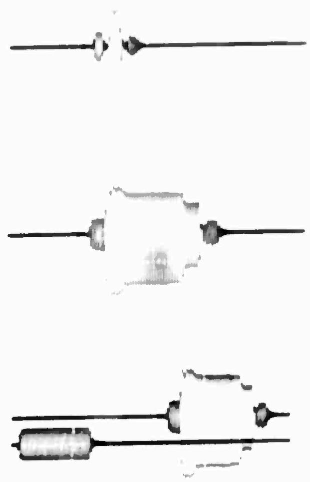
FIGURE 35

4 Microsec./Div.

4 Microsec./Div.

11 Microsec./Div.

FIGURE 36



4 Microsec./Div.

4 Microsec./Div.

5 Microsec./Div.

FIGURE 37

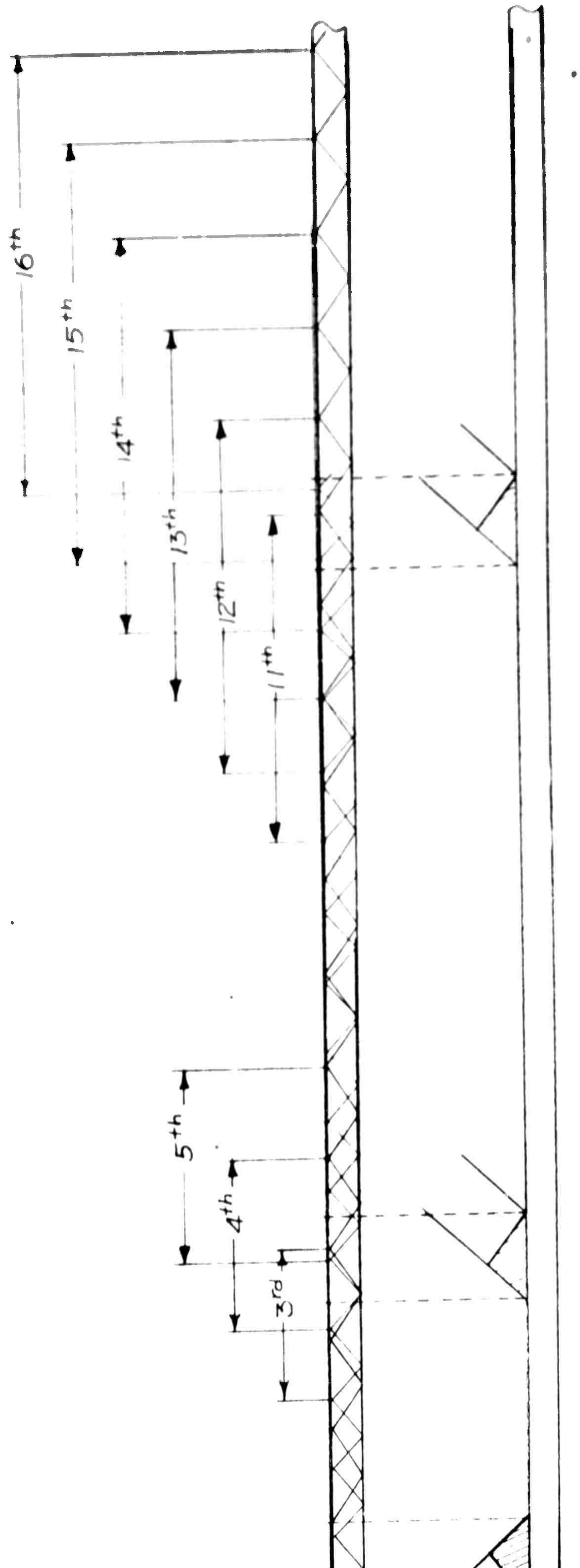
diameter transducer, it is found to be 6.13°. Then if the largest and smallest angles of refraction in steel are computed for an axial angle of incidence of 40°, they are found to be 60.2° and 41.9° respectively. The angle of refraction of the axial ray is found to be 50.5°. When an attempt was made, using these angles, to make a graphical construction which would explain the phenomena shown in Figures 36 and 37 it was not successful. However, it was found that if a smaller angle of beam divergence is assumed, Figures 36 and 37 are well explained by beam divergence. The smaller angle of beam divergence does not actually constitute an anomaly because, as explained above, the formula used is not strictly applicable. The resulting scale drawing is Figure 38. This figure shows the two extreme rays which were found graphically to be 3.8° on either side of the axial ray which is 50.5° from the normal.

Each reflection of the beam takes place over an area of the surface of the plate which is bounded by an ellipse. The larger the number of times the beam has been reflected, the longer is the major axis of the ellipse. In Figure 38, the distances which have been marked 3rd, 4th, 11th, etc. are the lengths of the major axes of the ellipses bounding the 3rd, 4th, 11th, etc., reflections respectively. It may be seen from the figure that when the separation between transducer supports is 10.9 cm. the receiving transducer covers parts of the 3rd, 4th, and 5th reflections resulting in the three pulses of Figure 37. When the separation between transducer supports is 25.0 cm., the receiving transducer covers parts of the 11th, 12th, 13th, 14th, 15th, and 16th reflections resulting in the six pulses of Figure 36.

Thus it was possible to explain Figures 36 and 37 in terms of beam divergence and the angle from the axis to the extreme rays was found to be 3.8°.

3.6 Experimental Verification of the Period Equation

A check was made using Worlton's curves of phase velocity versus frequency-thickness product for aluminum to make sure that the experimental techniques of the authors were equivalent to his in spite of being somewhat different. The 10 mc. coil and transducers and the 35° wedges (giving a trace velocity of 185,000 in./sec.) were used with the .0502 in. plate. A mode was found at 7.72 mc. which, in terms of frequency-thickness product, is $7.72 \times 10^6 \times .0502 = 3.87 \times 10^5$ in.-cycle/sec. This proved to be the third symmetrical mode. The fourth symmetrical and fourth asymmetrical modes also checked. Since Worlton has verified more than 50 points on these curves, this check was thought to be adequate.



BEAM DIVERGENCE IN A 1/4 INCH PLATE   SCALE 3:4

FIGURE 38

3.7 Measurement of Group Velocity

There is an inherent uncertainty in the measurement of group velocity. Since Lamb waves are dispersive, the different components of the pulse travel with different velocities. Consequently the transmitted and received pulses do not have the same shape. Therefore, some arbitrary criterion must be established for deciding at what point in time a given pulse occurs. In the following, the time at which a pulse occurred was taken to be the time at which its leading edge reached 50% of the maximum voltage attained.

The method used for measuring group velocities was to set the transmitting and receiving transducers close together and measure the distance and delay time; the transmitting and receiving transducers were then moved further apart, and the distance and delay time were again measured. The difference between the two distances divided by the difference between the two delay times is the group velocity.

The results of the group velocity measurements are shown in Table I along with the values obtained form the computed curves (Figure 15) The variance due to experimental error in these measurements of group velocity was computed by first computing the estimate for each of the seven pairs of measurements using the formula

$$\sigma^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{(n-1)}$$

where, in this case, $\bar{x}$ is the average group velocity and n = 2. Seven such values were thus obtained which were averaged to obtain the estimate of the variance due to experimental error.

The variance of sample averages for samples of size two will be one-half of this variance, and the corresponding standard deviation of the sample averages due to experimental error was found to be 659 in./sec. The calculated values of group velocity for the first and second symmetrical modes agree with the average experimental values within three standard deviations. The calculated value for the fifth symmetrical mode does not agree quite so well; however V_L/V_S for steel is actually 1.845 rather than 1.8 and the rate of change of group velocity with respect to frequency-thickness product is fairly large for the 5th mode at a frequency-thickness product of $.934 \times 10^5$ in.-cycle/sec so that a slight shift of the curves would cause a relatively large error in velocity. If another set of curves for $V_L/V_S = 1.85$ were available it would be possible to interpolate and obtain an accurate calculated value.

TABLE I

Plate Thick-ness, in.	Frequency, mc.	10^{-5} Frequency x Thickness, in.-cycle/sec.	Trace Velocity in./sec.	Mode	Group Velocity, in./sec. First Replication	Second Replication	Average	Calculated Value, (V_L/V_S-1.8)
1/32	2.46	0.769	408,000	2-A	129,500	128,100	128,800	
1/32	3.28	1.025	408,000	2-S	73,300	71,800	72,550	---
1/32	2.75	0.859	308,500	2-A	138,800	138,300	138,550	
1/8*	2.35	2.94	308,500		103,700	101,900	102,800	
1/8	3.14	3.925	308,500	5-S	76,000	74,000	75,000	70,500
1/32	2.99	0.934	164,100	1-S	70,100	69,700	69,900	71,200
1/32	9.13	2.85	164,100	2-S	90,600	90,500	90,650	88,900

* It is doubtful that the observed signal was actually a Lamb wave.

Standard Deviation Due to Experimental Error of Sample Averages 659 in./sec.

The data of Table II were taken at a latter date than the data of Table I and consequently are tabulated separately. The standard deviation of the sample averages due to experimental error was computed as described for Table I and was found to be 925 in./sec. Of the eight values of calculated group velocity which could be read from Figure 15, five do not agree with the corresponding experimental value within three standard deviations. The most likely explanation of the discrepancies is again the fact that the ratio of longitudinal to shear velocity for steel is not 1.8. In the worst of these eight cases, the discrepancy is 11%.

Measurements were made on thicker plates as well, but it was determined that the modes of vibration in the thicker plates were definitely not Lamb waves.

In order to study the first asymmetrical mode, the 1/4" polystyrene wedge described previously was used. The wedge angle was 60°; this gives a trace velocity of 1.027×10^5 in./sec. A rough measurement showed the group velocity to be 1.5×10^5 in./sec. at the frequency-thickness product used. This seemed, at first, to be unusual, since for all other modes the phase velocity always exceeded the group velocity. However, there is no reason why group velocity cannot exceed phase velocity. In this case of the first asymmetrical mode at a phase velocity of 1.027×10^5 in./sec., the value of group velocity calculated from data taken from Worlton's curves for stainless steel is 1.2×10^5 in./sec. This agrees closely enough considering that only a rough measurement was made.

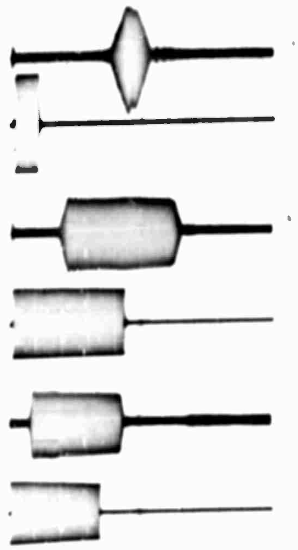
3.8 Some Atypical Waveforms

Figure 39 is typical of the pulses received. Most of the other photographs described in this section were taken because the waveforms depicted were more or less anomalous. In Figure 40 is shown the waveform resulting from exciting the second symmetrical mode in the 1/32" steel plate using a 15° wedge at 3.44 mc. The upper and lower exposures show the pulse received when the plate was excited with a 15 microsecond pulse, and the middle exposure shows the pulse received when the plate was excited with a 2.5 microsecond pulse. It is easily seen from Figure 14 that the curve of phase velocity versus frequency-thickness product is vertical for these values (phase velocity = 4.08×10^5 in./sec., fd = 1.075×10^5 cycle-in./sec.) and that consequently the variation of phase velocity over the spectrum will be very large so that one can expect an extremely large amount of distortion, as is evident in Figure 40. As usual, the lower trace in the middle and lower exposures is the wave meter output.

TABLE II

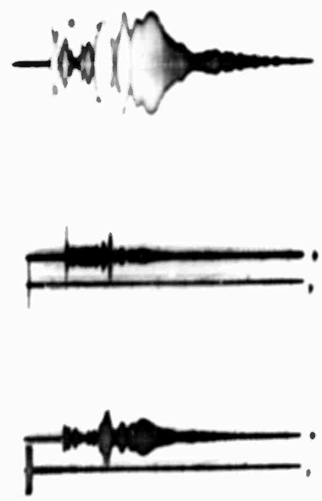
Plate Thick-ness, in.	Frequency, mc.	10^{-5} Fre-quency x Thickness, in -cycle/sec.	Trace Velocity in./sec.	Mode	Group Velocity, in./sec. First Repli-cation	Second Repli-cation	Average	Calculated Value, (V_L/V_S = 1.8)
0.0330"	4.95	1.632	228,000	2-S	188,500	189,000	188,750	195,200
0.0330"	5.40	1.781	228,000	2-S	190,500	191,000	190,750	190,000
0.0330"	8.70	2.87	233,800	3-S	180,700	181,100	180,900	194,000
0.0330"	8.70	2.87	233,800	3-A	140,000	140,000	140,000	
0.0330"	10.00	3.30	233,800	3-S	178,000	178,700	178,350	180,000
0.0330"	12.05	3.98	233,800	4-S	177,300	178,300	177,800	196,800
0.0330"	12.05	3.98	233,800	4-A	145,200	145,200	145,200	
0.0330"	12.40	4.09	233,800	4-S	178,600	183,800	181,200	196,800
0.0330"	12.40	4.09	233,800	4-A	143,300	145,700	144,500	
0.0330"	13.40	4.42	233,800	4-S	189,200	189,000	189,100	192,900
0.0330"	13.00	4.29	233,800	4-S	185,800	184,200	185,000	196,800

Best Estimate of Standard Deviation of Sample Averages Due to Experimental Error 925 in./sec.



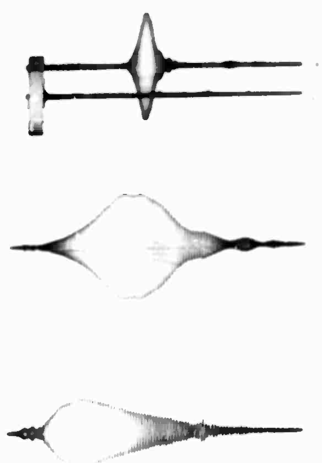
U. 10 Microsec. Div.
M. 20 Microsec. Div.
L. 50 Microsec. Div.

FIGURE 39



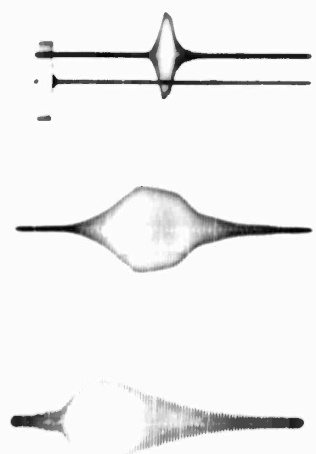
U. 40 Microsec. Div.
M. 50 Microsec. Div.
L. 50 Microsec. Div.

FIGURE 40



U. 20 Microsec. /Div.
M. 4 Microsec. /Div.
L. 4 Microsec. /Div.

FIGURE 41



U. 20 Microsec. /Div.
M. 4 Microsec. /Div.
L. 4 Microsec. /Div.

FIGURE 42

In Figure 41 is shown the waveform resulting from exciting the second asymmetrical mode in the 1/32" steel plate. The trace velocity was 4.08×10^5 in./sec. and the frequency-thickness product was $.769 \times 10^5$ in.-cycles/sec. In the upper and middle exposures the driving pulse was of 15 microseconds duration, and the lower exposure the driving pulse was of 2.5 microseconds duration. It is very evident that the 2.5 microsecond pulse has been stretched to about ten times its original duration. It was suspected that this effect might be due to reverberation within the plastic wedge. That this is not the case is shown by Figure 42 which was taken under the same conditions as Figure 41 except that the wedge was made as shown in Figure 44 instead of as shown in Figure 43 and the attenuation was decreased to compensate for the added lucite. Figure 45 is the same as Figure 40 except that the trace velocity was increased to 6.08×10^5 in./sec. by using a 10° wedge instead of the 15° wedge used when taking Figure 40.

Figure 46 was taken using the 1/32" plate, a trace velocity of 6.08×10^5 in./sec., and a frequency of 2.72 mc. In the upper and middle exposures, the driving pulse was 15 microseconds; in the lower exposure, the duration of the driving pulse was 2.5 microseconds. This appears to be a clearly defined mode but it falls between the curves for the first symmetrical and first asymmetrical modes.

Figure 47 was taken using the 1/32" plate, a trace velocity of 6.08×10^5 in./sec., and a frequency of 2.34 mc. The driving pulse was 15 microseconds in the upper exposure and 2.5 microseconds in the middle and lower exposures. It appears that this may be the second asymmetrical mode. The received waveform when the plate was excited with a 2.5 microsecond pulse was actually stretched out over a longer period of time that when the plate was excited with a 15 microsecond pulse. Possibly, the wide spectrum of the 2.5 microsecond pulse was exciting more than one mode.

The pulsed oscillator was connected to the external gate pulse generator so that a longer pulse could be obtained. Figures 39 and 48 show the effect of pulse length on the waveform of a received Lamb wave. The upper exposure of Figure 39 was taken with a pulse length of 10 microseconds, the middle exposure was taken with a pulse length of 90 microseconds, and the lower exposure was taken with a pulse length of 170 microseconds. The upper exposure of Figure 48 was taken with a trace velocity of 1.03×10^5 in./sec. (60° polystyrene wedge) at a frequency-thickness product of $.712 \times 10^5$ cycle-in./sec. (1/32" steel plate). This

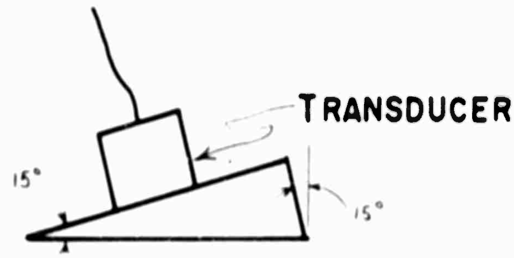


FIGURE 43

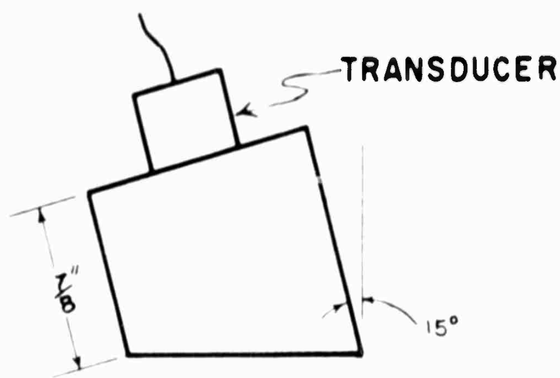
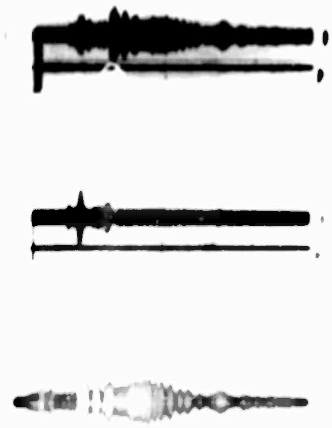
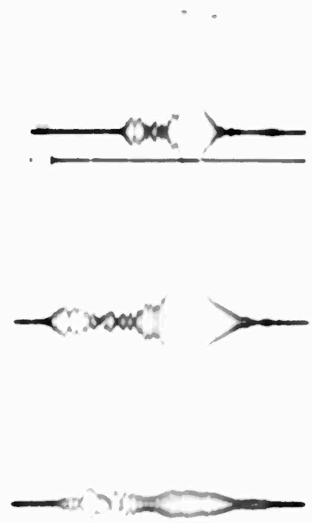


FIGURE 44



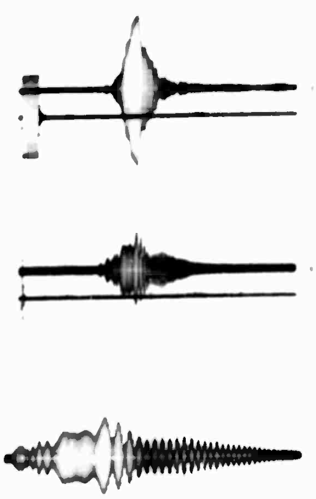
U. 50 Microsec. Div.
M. 50 Microsec. Div.
L. 40 Microsec. Div.

FIGURE 45



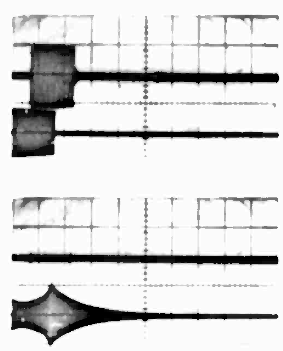
U. 20 Microsec./Div.
M. 10 Microsec./Div.
L. 10 Microsec./Div.

FIGURE 46



U. 20 Microsec./Div.
M. 20 Microsec./Div.
L. 5 Microsec./Div.

FIGURE 47



U. 100 Microsec./Div.

L. 100 Microsec./Div.

FIGURE 48

is the first asymmetrical mode. This pulse passed through several maxima and minima as the frequency was varied; this was attributed to the shape of the frequency spectrum. The second exposure of Figure 48 was taken at a trace velocity of $1.03 \times 10^{+5}$ in./sec. and frequency-thickness product of 1.03×10^5 cycle-in./sec., and shows that there is essentially no transmission of sound through the plate at this frequency-thickness product and trace velocity. (The lower waveforms are the output of the wavemeter.)

3.9 Unexplained Modes

An anomalous mode was noticed while using the 60° wedges with the 1/8" plate in the 3 mc. region. This is shown in Figure 49. The upper exposure was taken at 2.34 mc., the middle exposure was taken at 2.74 mc., and the lower exposure was taken at 3.32 mc. The incident angle exceeds the critical angle for both shear and longitudinal waves, so that these pulses are not the result of multiply reflected shear waves. The frequency-thickness products are thought to be too great for the first asymmetrical mode to be excited with this trace velocity. Figure 50 is similar except that the plate used was 1/4" thick, the frequency was held constant at 3.3 mc. and the receiving wedge was varied. The upper exposure of Figure 50 was taken with a 15° receiving wedge, the middle exposure was taken with a 25° receiving wedge, and the lower exposure was taken with a 35° receiving wedge. The transmitting wedge was 60° in each case. (Lower waveforms are the output of the wavemeter.)

It was also found that when a 35° wedge was used on the receiver and wedges with angles of 10°, 20°, and 45° were used on the transmitter, it was always possible to obtain a pulse at the receiver. The amplitude of the pulse received using unmatched wedges was about 1/5 of the amplitude received when both the transmitting and receiving wedges had angles of 35°.

3.10 Experiment on Slotted Plates

3.10.1 Objectives of the Slotted Plate Experiment

In order to obtain data which might indicate possible applications of ultrasonic lamb waves in the nondestructive testing of sheet and plate metals, an experiment was conducted using plates having highly idealized flaws. The primary objectives of this experiment were to determine whether or not a highly idealized flaw would have a significant

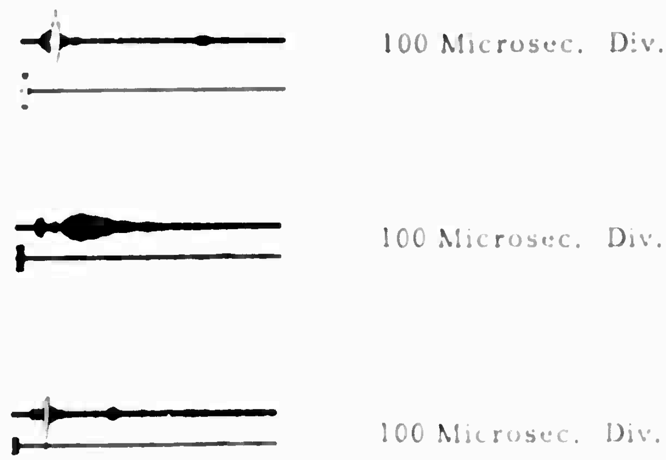


FIGURE 49

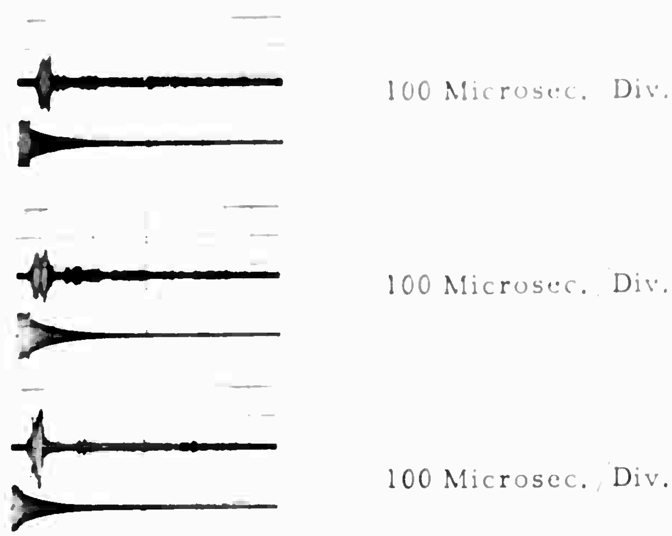


FIGURE 50

amount of insertion loss, and to attempt to correlate the amount of insertion loss with the size of the flaw.

3.10.2 Description of the Slotted Plate Experiment

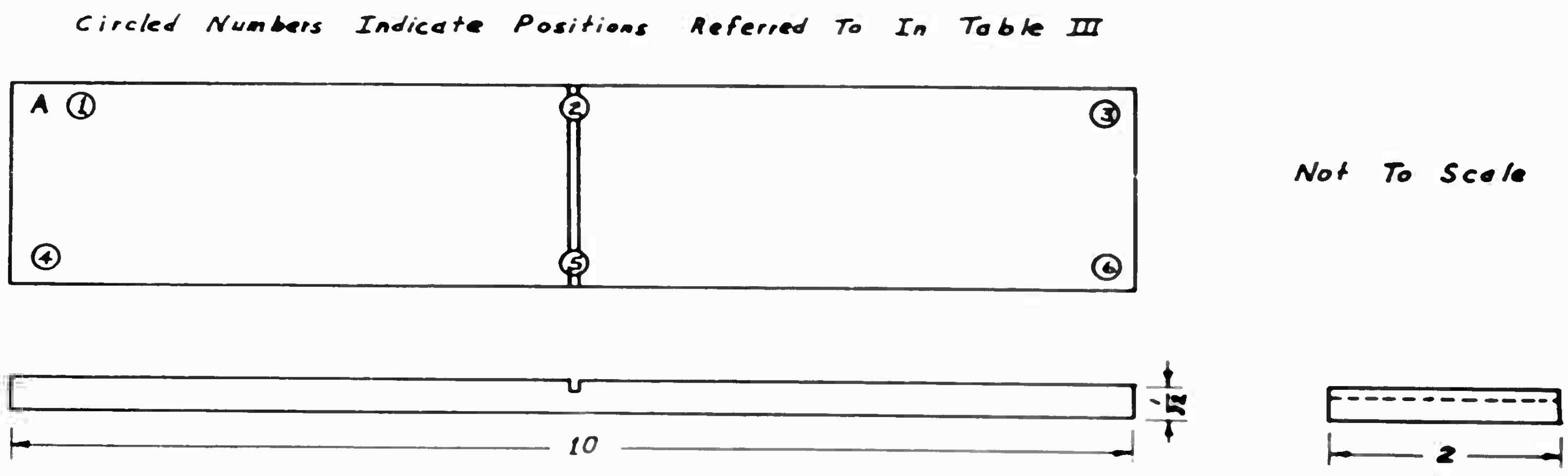
The test plates were of steel and consisted of 2" × 10" × 1/32" thick pieces of precision ground flat stock, Starrett type No. 496. The simulated flaws consisted of various depths of saw cuts, 1/32" wide. A typical plate with its saw cut is shown in Figure 51. Nine such test plates were prepared from one 10" × 18" × 1/32" steel plate. Saw cuts which varied in depth by increments of 0.003" were made in seven of these plates, and two were left without saw cuts to serve as reference plates. Letters were painted on the plates for identification as follows:

Plate Designation	Depth of Saw Cut
A	0.003"
B	0.006"
C	0.009"
D	0.012"
E	0.015"
F	0.018"
G	0.021"
H	0.000"
I	0.000"

The thickness of each plate was measured at each of the six positions shown in Figure 51. These thickness measurements are given in Table III.

3.10.3 Insertion Loss of Saw Cuts

The experiment performed on these plates consisted of comparing the transmission of Lamb waves through plates without saw cuts with the transmission through plates with saw cuts. In general, the effect of the saw cut was to alter the shape and power level of the received pulse. The change in power level was measured by introducing enough attenuation with the precision attenuator so that the signal level at the input to



TEST PLATE

FIGURE 51

TABLE III

Plate	Position	Thickness	Ave. Thickness
A	1	0.0334"	
	2	0.0326	
	3	0.0335	
	4	0.0334	
	5	0.0330	
	6	0.0329	0.0331
B	1	0.0334	
	2	0.0328	
	3	0.0330	
	4	0.0334	
	5	0.0325	
	6	0.0327	0.0330
C	1	0.0339	
	2	0.0330	
	3	0.0337	
	4	0.0326	
	5	0.0337	
	6	0.0335	0.0334
D	1	0.0327	
	2	0.0330	
	3	0.0334	
	4	0.0326	
	5	0.0329	
	6	0.0333	0.0330
E	1	0 0329	
	2	0.0330	
	3	0.0333	
	4	0.0325	
	5	0.0331	
	6	0.0332	0.0330
F	1	0.0326	
	2	0.0331	
	3	0.0335	
	4	0.0324	
	5	0.0330	
	6	0.0334	0.0330

TABLE III (Continued)

Plate	Position	Thickness	Ave. Thickness
G	1	0.0330	
	2	0.0329	
	3	0.0329	
	4	0.0334	
	5	0.0329	
	6	0.0327	0.0330
H	1	0.0330	
	2	0.0326	
	3	0.0325	
	4	0.0330	
	5	0.0328	
	6	0.0325	0.0327
I	1	0.0327	
	2	0.0327	
	3	0.0334	
	4	0.0329	
	5	0.0328	
	6	0.0333	0.0330

the oscilloscope was 0.18 volts peak-to-peak. (If the pulse shape was distorted so that the top of the pulse was not flat, the 0.18 volts was measured across the average envelope of the pulse). This amount of attenuation was recorded so that, for example, a signal recorded as 45 db was at a higher power level than a signal recorded as 44 db.

In order to minimize the experimental error a standard technique was adopted for obtaining acoustic coupling from the lucite wedges to the plate. This procedure consisted of applying a spot of oil the size of a penny beneath each wedge, applying enough pressure so that some oil was squeezed from beneath each wedge and wiping off the excess in front of the wedges. In order to determine the number of readings to average so that experimental results would be known within approximately ± 1 db, forty readings were taken using this technique. A frequency histogram of these forty readings is shown in Figure 52 and indicates that the experimental error has a normal distribution. Thus 99.7% of all measurements made using this technique should fall within plus or minus three standard deviations. The variance of the sample was computed from the formula

$$s^2 = \frac{1}{n} \sum_{i=1}^{k} f_i(x_i - \bar{x})^2$$

An unbiased estimate of the variance of the parent population is $n/(n-1)$ times the variance of the sample and this was found to be 1.05 db^2 so that the standard deviation of the experimental error is the square root of this or 1.024 db. Thus 99.7% of all measurements should fall within ± 3.072 db of the correct value. Better results can be obtained by taking a number of readings and then averaging them. The standard deviation of sample averages is $1/\sqrt{n}$ times the standard deviation of individual measurements; so that in this case it was necessary to take ten measurements and average them in order that the result would be known to ± 1 db.

The procedure followed in this experiment was to measure the level of the signal received from a plate with a saw cut and then repeat the measurement on a plate without a saw cut. This was repeated ten times and the difference was taken between the two averages. This difference is the insertion loss of the saw cut. A photograph was taken of a typical wave form for each set of measurements. The resulting data are given in Table IV. The photographs are described on the same page as the corresponding measurements and appear on the following page. The attenuator setting at which each exposure was taken is given in each case.

Each mode was investigated at only one point on its curve of phase velocity versus frequency-thickness product, and the data obtained are therefore characteristic of the mode only in the neighborhood of this point.

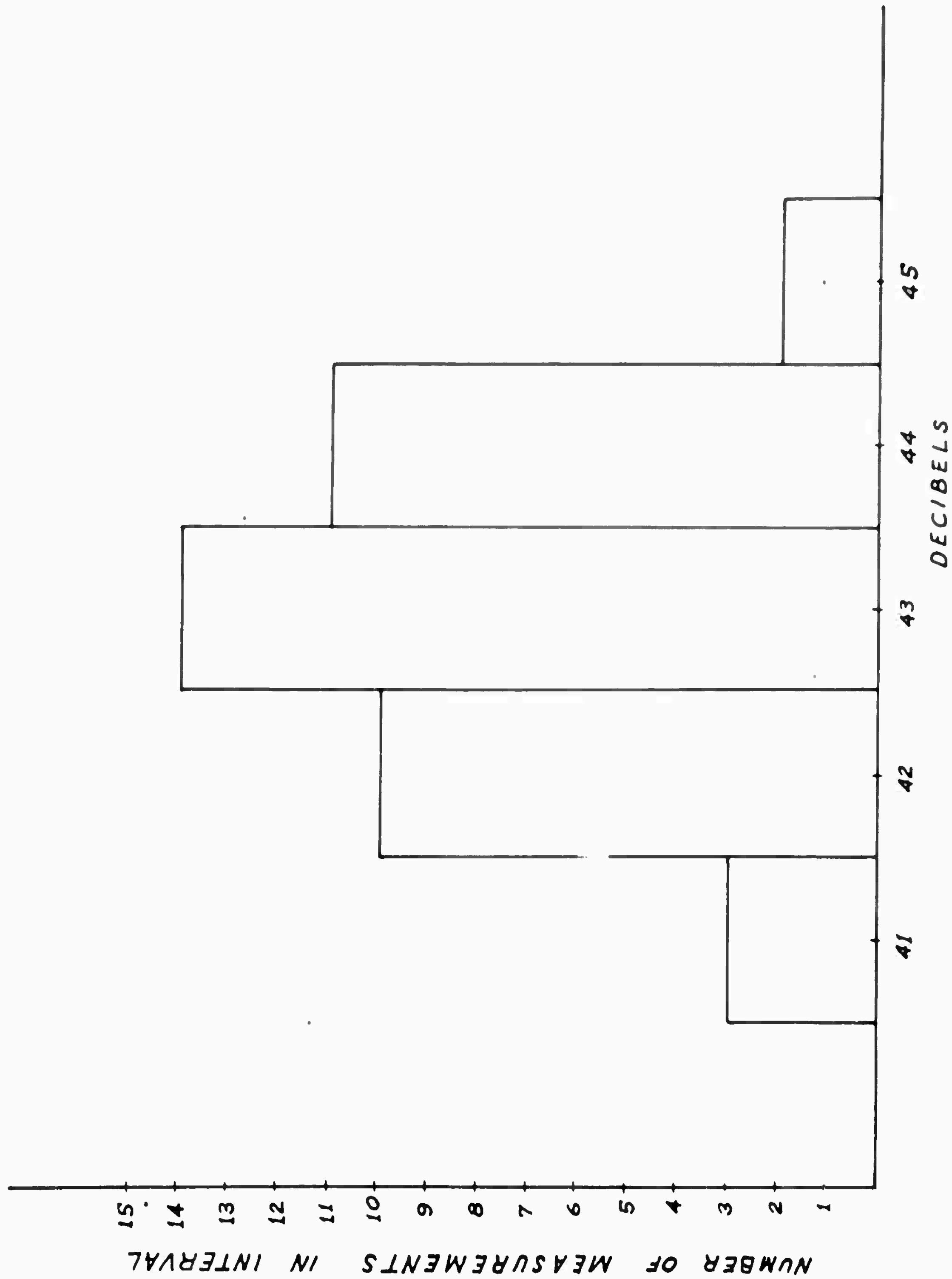

41
42
43
44
45
DECIBELS
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
NUMBER OF MEASUREMENTS IN INTERVAL

FIGURE 52

TABLE IV

Frequency = 2.93 mc | Wedge Angles = 40° | Wedge Material = Lucite | Orientation of cut = up | Mode in Ref. Plate = 1-S

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
I	40	40	42	43.5	41	43	43	45	40.5	44	42.20	
A	43	41	41	42	43	41	40.5	44	40.5	40	41.60	0.60
I	43	41	44.5	47	42.5	46	46	43.5	41	43	43.75	
B	39	46	39	41	44	40	49	49	42	47	43.6	0.15
I	41	44	40	42	40	39	38	38	41	45	40.8	
C	42	41	40	41	45	41	44	43	42	48	42.7	-1.9
I	39	46	42	41	47	43	43	43	44	43	43.1	
D	34	38	41	40	39	39	39	39	38	39	38.6	4.5

Figure 53
50 microsec./div.
U: Plate A, 44db
L: Plate I, 44db

Figure 54
50 microsec./div.
U: Plate B, 44db
L: Plate I, 44db

Figure 55
50 microsec./div.
U: Plate I, 42db
M: Plate C, 42db
L: Plate C, 44db

Figure 56
50 microsec./div.
U: Plate D, 39db
M: Plate D, 44db
L: Plate I, 44db

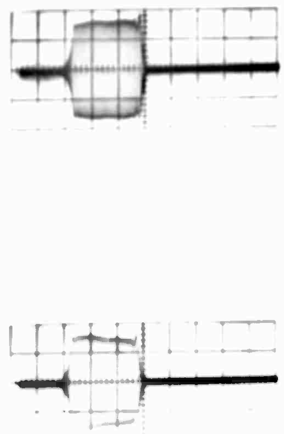


FIGURE 53

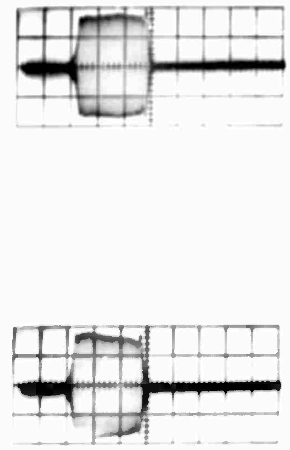


FIGURE 54

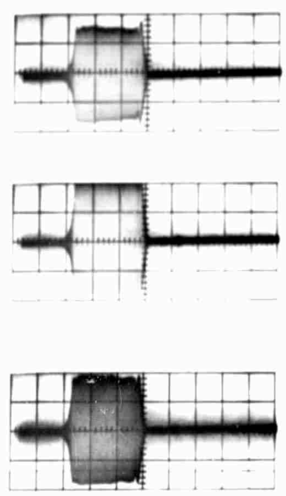


FIGURE 55

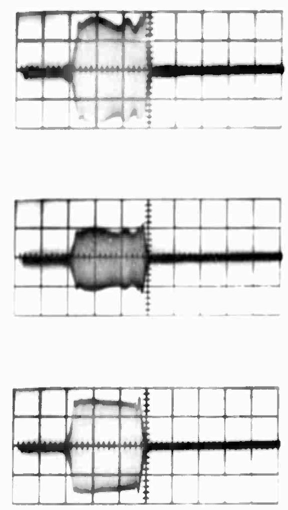


FIGURE 56

TABLE IV (Continued)

Frequency = 2.93 mc

Wedge Angle 40°
Wedge Material - Lucite

Orientation of cut = up
Mode in Ref. Plate = 1-S

Plate	Attenuator Setting, Decibels: Replication No. 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
I	44	43	44	44	43	40	43	43	43	41	42.8	
E	37	37	38	36	36	39	36	36	36	37	36.8	6.0
I	39	35	45	44	43	41	43	44	44	43	42.1	
F	35	30	36	39	34	36	35	37	37	33	35.2	6.9
I	44	44	44	45	46	43	44	44	43	42	43.9	
G	40	40	41	40	40	42	40	39	39	40	40.1	3.8

Figure 57
50 microsec./div.
U: Plate E 39db
M: Plate E, 43db
L: Plate I, 43db

Figure 58
50 microsec./div.
U: Plate I, 44db
M. Plate F, 44db
L: Plate F, 33db

Figure 59
50 microsec./div.
U: Plate I, 44db
M: Plate G, 44db
L: Plate G, 41db

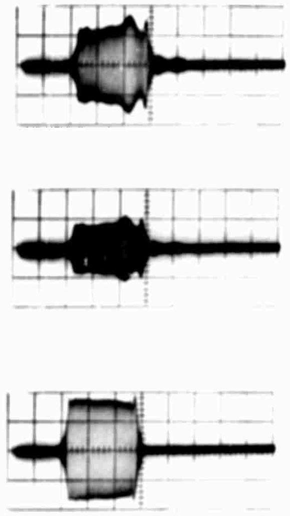


FIGURE 57

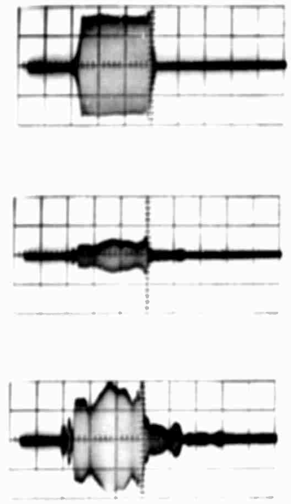


FIGURE 58

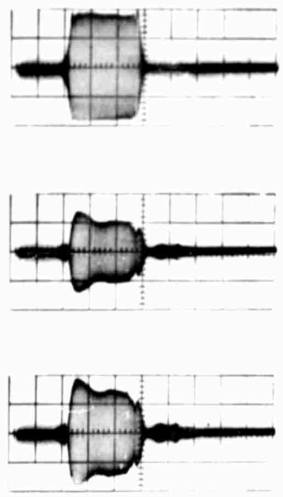


FIGURE 59

TABLE IV (Continued)

Frequency = 9.15 mc | Wedge Angles = 40, Wedge Material = Lucite | Orientation of cut = up, Mode in Ref Plate = 2-S

Plate	Attenuator Setting, Decibels — Replication No. 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
I	46	46	43	47	43	41	43	47	47	44	44.7	
A	37	39	38	37	36	36	40	38	38	40	37.9	6.8
I	48	46	47	48	47	46	48	46	48	44	46.8	
B	34	33	32	30	26	34	30	30	35	27	31.1	15.7
I	42	47	44	44	43	44	42	39	40	40	42.5	
C	42	43	43	37	41	41	38	39	38	34	39.6	2.9
I	43	43	49	42	41	43	45	42	38	39	42.5	
D	33	26	23	36	30	34	26	26	36	31	30.1	12.4

Figure 60
20 microsec./div.
U: Plate I, 44db
M: Plate A, 44db
L: Plate A, 40db

Figure 61
20 microsec./div.
U: Plate B, 27db
M: Plate B, 46db
L: Plate I, 46db

Figure 62
50 microsec./div.
U: Plate I, 41db
M: Plate C, 41db
L: Plate C, 38db

Figure 63
20 microsec./div.
U: Plate I, 39db
M: Plate D, 39db

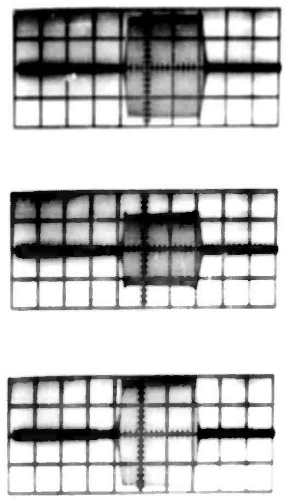


FIGURE 60

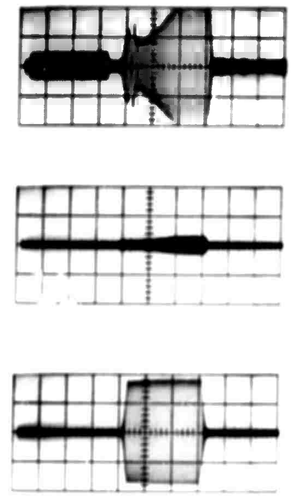


FIGURE 61

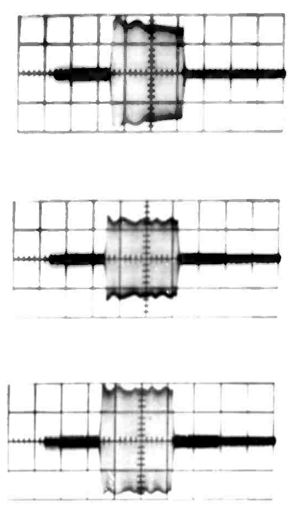


FIGURE 62

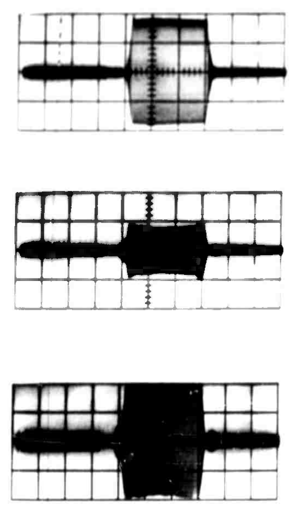


FIGURE 63

TABLE IV (Continued)

Frequency = 9.15 mc					Wedge Angles = 40° Wedge Material = Lucite						Orientation of cut = up Mode in Ref. Plate = 2-S	
Plate	Attenuator Setting, Decibels										Average	Insertion Loss
	Replication No.											
	1	2	3	4	5	6	7	8	9	10		
I	44	41	47	42	48	45	39	41	45	44	43.6	
E	37	40	39	39	41	36	39	40	40	36	38.7	4.9
I	45	45	46	46	45	45	43	44	44	43	44.6	
F	32	34	35	32	32	32	28	31	31	28	31.5	13.1
I	42	41	43	42	39	38	42	40	39	41	40.7	
G	30	31	29	28	27	25	28	28	26	27	27.9	12.8

Figure 64
20 microsec./div.
U: Plate I, 43db
M. Plate F, 43db
L: Plate F, 28db

Figure 65
20 microsec./div.
U: Plate I, 41db
M. Plate G, 41db
L. Plate G, 27db

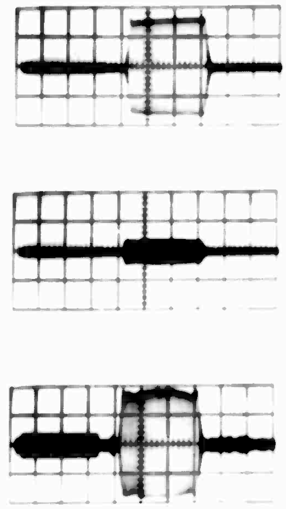


FIGURE 64

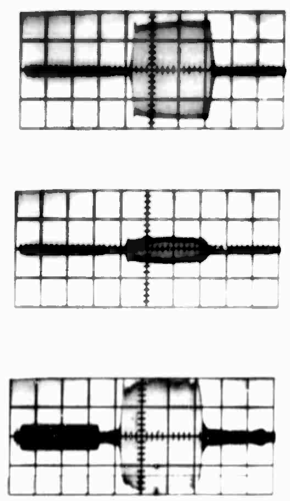


FIGURE 65

TABLE IV (Continued)

Frequency = 10.7 mc | Wedge Angle = 27.5° | Wedge Material = Lucite | Orientation of cut = up | Mode in Ref. Plate = 3-S

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
I	44	43	43	42	44	43	44	43	45	44	43.5	
A	41	41	39	39	38	39	41	39	40	39	39.6	3.9
I	44	43	44	42	43	44	43	44	43	42	43.2	
B	39	42	38	30	41	34	40	41	40	38	38.3	4.9
I	41	40	40	42	39	43	41	42	42	42	41.2	
C	41	42	42	40	41	41	39	43	45	44	41.8	-0.6
I	41	43	43	39	44	42	41	42	43	43	42.1	
D	38	38	38	38	37	39	37	40	37	39	38.1	4.0

Figure 66	Figure 67	Figure 68	Figure 69
10 microsec./div.	10 microsec./div.	10 microsec./div.	10 microsec./div.
U: Plate I, 44db	U: Plate B, 38db	U: Plate I, 42db	U: Plate I, 43db
M: Plate A, 44db	M: Plate B, 42db	M: Plate C, 44db	M: Plate D, 43db
L: Plate A, 39db	L. Plate I, 42db	L: Plate C, 44db	L: Plate D, 39db

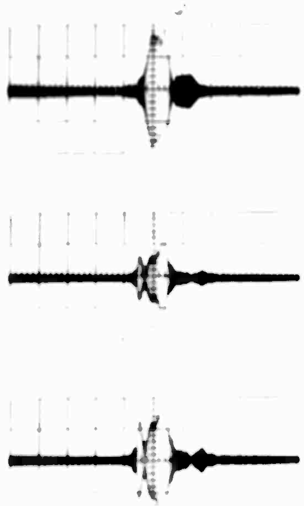


FIGURE 66

FIGURE 67

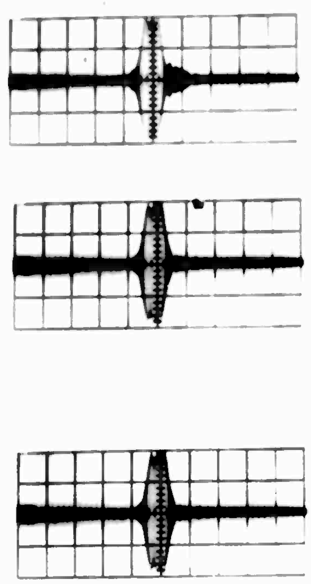


FIGURE 68

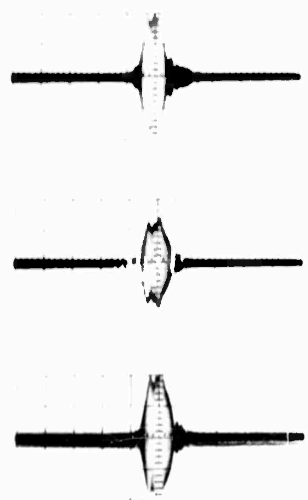


FIGURE 69

TABLE IV (Continued)

Frequency = 10.7 mc | Wedge Angle = 27.5° | Wedge Material = Lucite | Orientation of cut = up | Mode in Ref. Plate = 3-S

Plate	Attenuator Setting, Decibels										Average	Insertion Loss
	Replication No.:											
	1	2	3	4	5	6	7	8	9	10		
I	45	43	43	39	41	40	41	41	42	40	41.5	
E	36	30	29	37	23	35	31	35	26	34	31.6	9.9
I	41	42	41	43	42	43	41	42	41	41	41.7	
F	34	36	31	31	37	39	36	35	32	38	34.9	6.8
I	38	42	38	40	37	42	41	40	40	42	40.0	
G	37	36	34	32	34	35	33	33	35	33	34.2	5.8

Figure 70
10 microsec./div.
U: Plate I, 40db
M: Plate E, 40db
L: Plate E, 34db

Figure 71
10 microsec./div.
U: Plate I, 41db
M: Plate F, 41db
L. Plate F, 38db

Figure 72
10 microsec./div.
U: Plate I, 42db
M: Plate G, 42db
L: Plate G, 33db

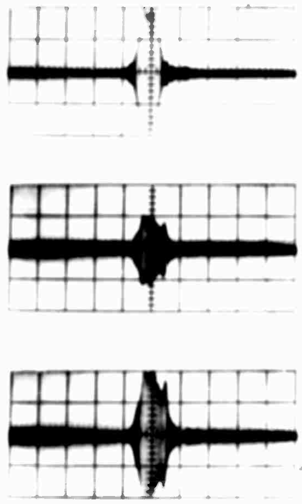


FIGURE 70

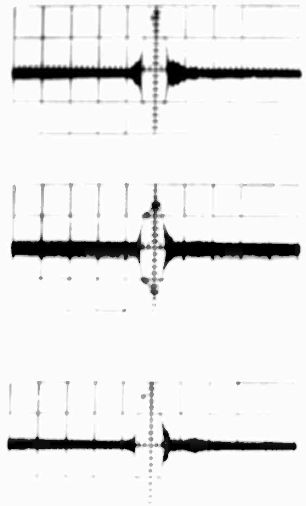


FIGURE 71

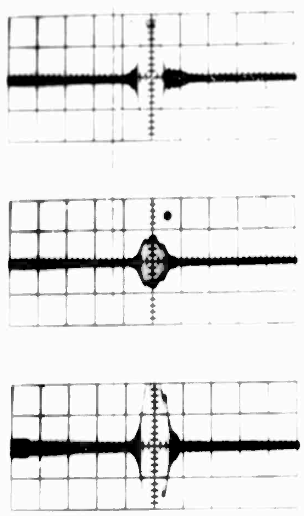


FIGURE 72

TABLE IV (Continued)

Frequency = 9.55 mc

Wedge Angles = 30°
Wedge Material = Lucite

Orientation of cut = up
Mode in Ref. Plate = 3-A

Plate	Attenuator Setting, Decibels										Average	Insertion Loss
	Replication No.											
	1	2	3	4	5	6	7	8	9	10		
I	46	47	46	44	47	49	48	48	48	49	47.2	
A	40	41	40	39	43	44	45	43	42	43	42.0	5.2
I	47	47	46	45	47	46	48	49	49	48	47.2	
B	41	38	36	35	40	38	38	42	42	37	38.7	8.5
I	49	45	48	48	48	50	48	50	49	48	48.3	
C	43	44	42	42	44	43	44	42	42	43	42.9	5.4
I	47	48	48	49	48	50	52	51	50	50	49.3	
D	27	24	25	23	25	21	26	29	28	26	25.4	23.9

Figure 73
20 microsec./div.
U: Plate I, 48db
M: Plate A, 48db
L: Plate A, 43db

Figure 74
20 microsec./div.
U: Plate B, 42db
M: Plate B, 49db
L: Plate I, 49db

Figure 75
20 microsec./div.
U: Plate I, 48db
M: Plate C, 48db
L: Plate C, 43db

Figure 76
20 microsec./div.
U: Plate D, 26db
M: Plate D, 50db
L: Plate I, 50db

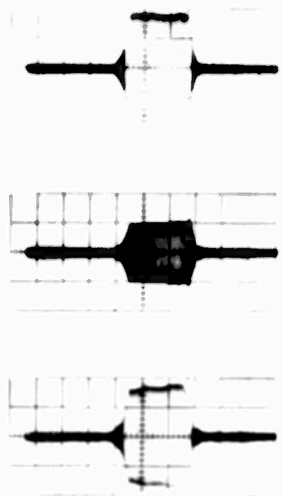


FIGURE 73

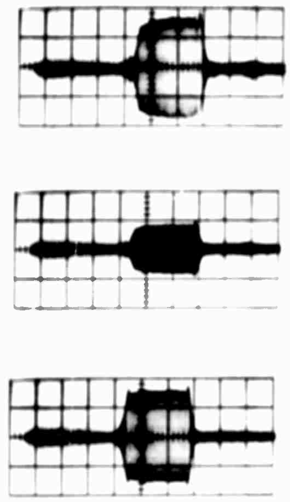


FIGURE 74

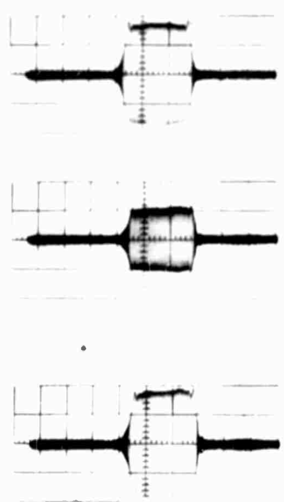


FIGURE 75

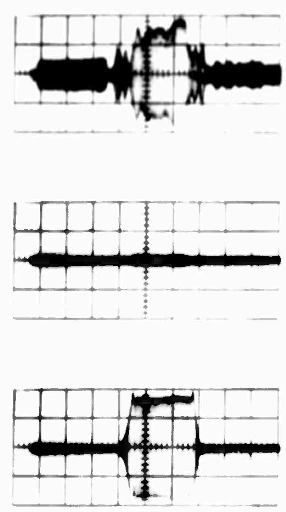


FIGURE 76

TABLE IV (Continued)

Frequency = 9 55 mc | Wedge Angle = 30° | Wedge Material = Lucite | Orientation of cut = up | Mode in Ref. Plate = 3-A

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
I	50	50	47	49	48	47	47	49	49	49	48.5	
E	38	37	38	38	36	38	38	40	38	39	38.0	10.5
I	48	50	49	49	50	48	47	49	49	49	48.8	
F	42	44	44	43	44	41	43	44	45	44	43.4	5.4
I	49	49	49	45	48	47	49	47	48	49	48.0	
G	36	37	37	37	36	36	36	37	37	36	36.5	11.5

Figure 77
20 microsec./div.
U: Plate I, 49db
M: Plate E, 49db
L: Plate E, 39db

Figure 78
20 microsec./div.
U: Plate I, 49db
M: Plate F, 49db
L: Plate F, 44db

Figure 79
20 microsec./div.
U: Plate I, 49db
M: Plate G, 49db
L: Plate G, 36db

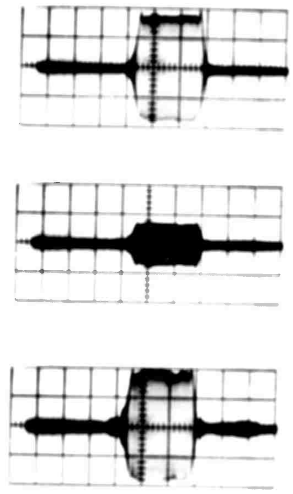


FIGURE 77

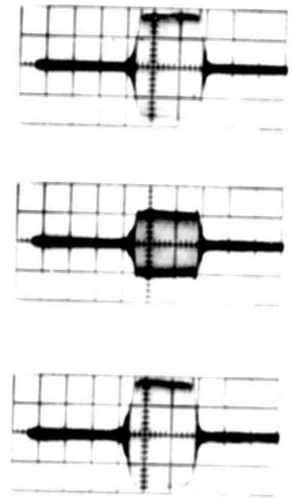


FIGURE 78

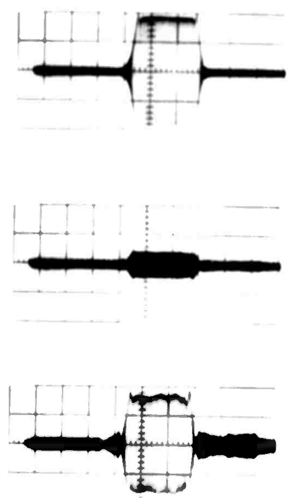


FIGURE 79

TABLE IV (Continued)

Frequency = 9.50 mc					Wedge Angle = 20°	Wedge Material = Lucite					Orientation of cut = up	Mode in Ref. Plate = 4-A
Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
I	47	43	45	43	46	47	45	44	43	47	45.0	
A	42	46	43	45	46	43	44	45	45	45	44.4	0.6
I	41	44	43	41	43	45	42	43	45	45	43.2	
B	44	40	41	42	40	41	38	43	41	43	41.3	1.9
I	42	45	45	45	44	44	45	43	44	44	44.1	
C	28	20	34	26	24	23	29	25	28	20	25.7	18.4
I	45	43	47	46	46	45	45	44	45	46	45.2	
D	28	30	29	29	29	26	25	29	30	28	28.3	16.9

Figure 80
10 microsec./div.
U: Plate I, 47db
M: Plate A, 47db
L: Plate A, 45db

Figure 81
10 microsec./div.
U: Plate I, 45db
M: Plate I, 45db
L: Plate B, 43db

Figure 82
10 microsec./div
U: Plate I, 44db
M: Plate C, 44db
L: Plate C, 20db

Figure 83
10 microsec./div.
U: Plate I, 46db
M: Plate D, 46db
L: Plate D, 28db

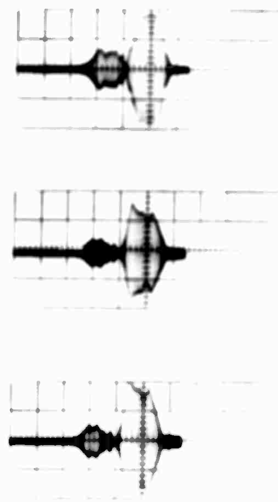


FIGURE 80

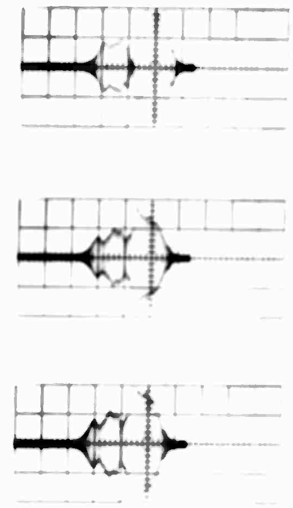


FIGURE 81

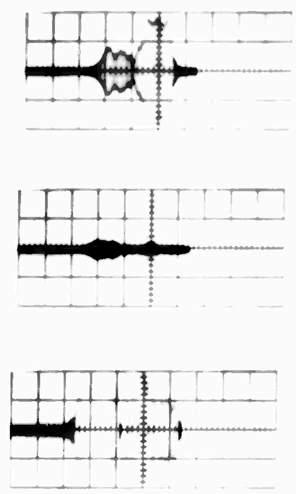


FIGURE 82

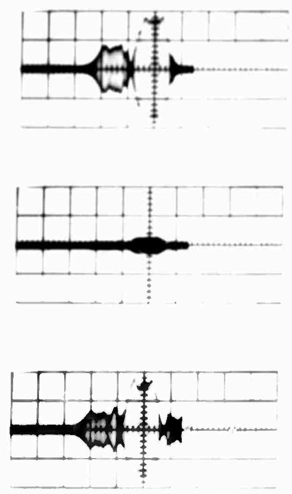


FIGURE 83

TABLE IV (Continued)

Frequency = 9.50 mc					Wedge Angle = 20° Wedge Material = Lucite						Orientation of cut = up Mode in Ref. Plate = 4-A	
Plate	Attenuator Setting, Decibels										Average	Insertion Loss
	Replication No.:											
	1	2	3	4	5	6	7	8	9	10		
H	41	47	46	49	51	49	45	44	51	46	46.9	
E	31	33	36	36	36	33	30	34	33	36	33.8	13.1

Plates F and G were not measured because the distortion was too great.

Figure 84	Figure 85	Figure 86
10 microsec./div.	10 microsec./div.	10 microsec./div.
U: Plate H, 46db	U: Plate H, 51db	U: Plate H, 49db
M: Plate E, 46db	M: Plate F, 51db	M: Plate G, 49db
L: Plate E, 36db	L. Plate F, 40db	L: Plate G, 34db

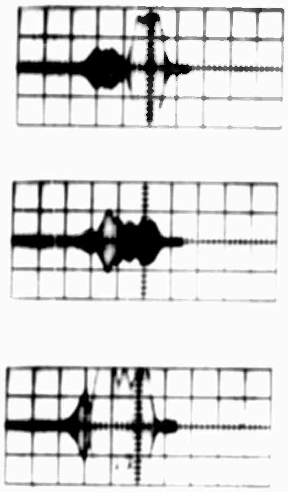


FIGURE 84

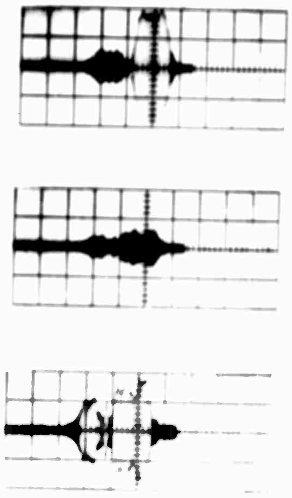


FIGURE 85

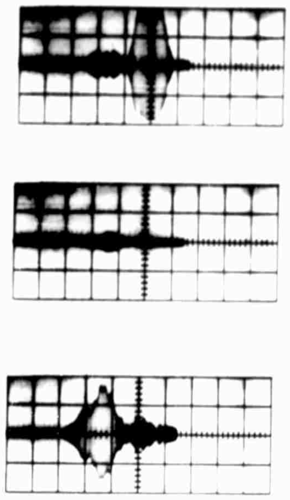


FIGURE 86

TABLE IV (Continued)

Frequency = 8.93 mc | Wedge Angle = 20° | Wedge Material = Lucite | Orientation of cut = up | Mode in Ref. Plate = 4-S-A

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
I	47	51	49	49	50	49	46	49	48	47	48.5	
A	36	38	38	39	37	34	36	35	33	33	35.9	12.6
H	47	44	44	48	46	46	44	44	44	44	45.1	
B	30	32	30	30	26	28	28	25	30	31	29.0	16.1
H	47	46	46	46	46	45	46	46	45	44	45.7	
C	40	36	34	35	36	37	32	34	31	36	35.1	10.6
H	47	47	46	47	46	49	47	47	48	47	47.1	
D	20	20	22	22	28	31	27	26	25	21	24.2	22.9

Figure 87	Figure 88	Figure 89	Figure 90
10 microsec./div.	20 microsec./div.	20 microsec./div.	20 microsec./div.
U: Plate H, 47db	U: Plate H, 44db	U: Plate H, 44db	U: Plate H, 46db
M: Plate A, 47db	M: Plate B, 44db	M: Plate C, 44db	M: Plate D, 46db
L: Plate A, 33db	L: Plate B, 31db	L: Plate C, 36db	L: Plate D, 22db

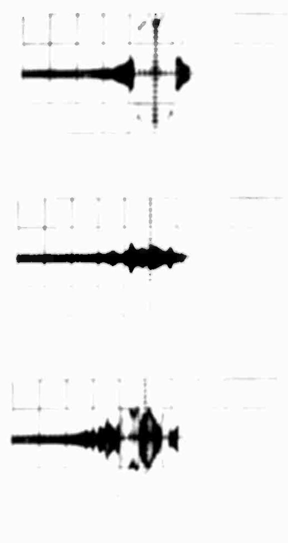


FIGURE 87

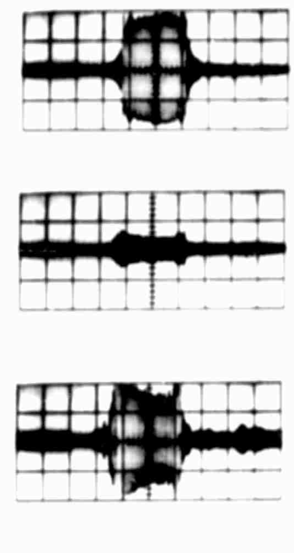


FIGURE 88

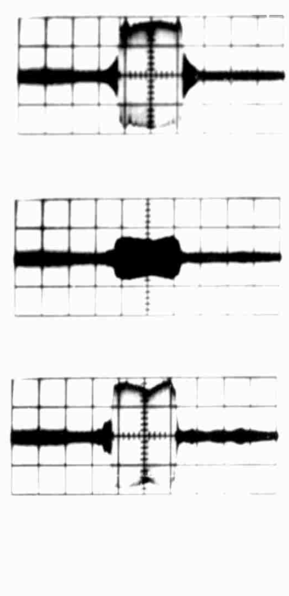


FIGURE 89

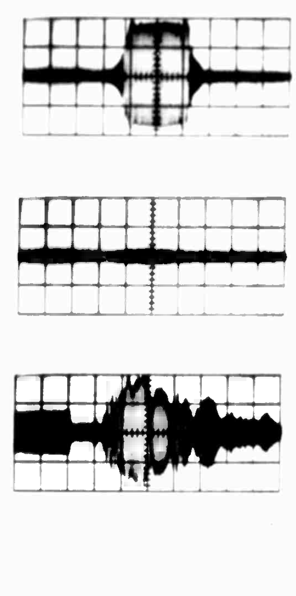


FIGURE 90

TABLE IV (Continued)

Frequency = 8.93 mc | Wedge Angle = 20° | Wedge Material = Lucite | Orientation of cut = up | Mode in Ref. Plate = 4-S-A

Plate	Attenuator Setting, Decibels, Replication No. 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
H	47	48	49	47	48	48	46	47	47	45	47.2	
E	31	39	39	33	39	38	33	38	36	38	36.4	10.8
H	45	47	45	46	46	47	45	47	47	47	46.2	
F	35	38	40	30	37	36	37	39	39	39	37.0	9.2
H	44	46	45	47	47	45	45	46	43	43	45.1	
G	29	26	25	23	22	25	25	20	23	26	24.4	20.7

Figure 91
20 microsec./div.
U: Plate H, 45db
M. Plate E, 45db
L: Plate E, 38db

Figure 92
20 microsec./div.
U: Plate H, 47db
M: Plate F, 47db
L: Plate F, 39db

Figure 93
20 microsec./div.
U: Plate H, 43db
M: Plate G, 43db
L: Plate G, 26db

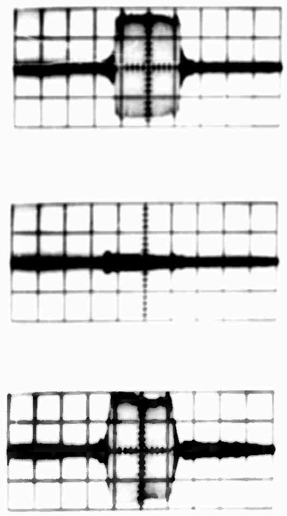


FIGURE 91

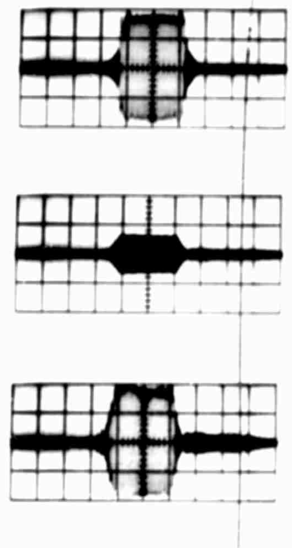


FIGURE 92

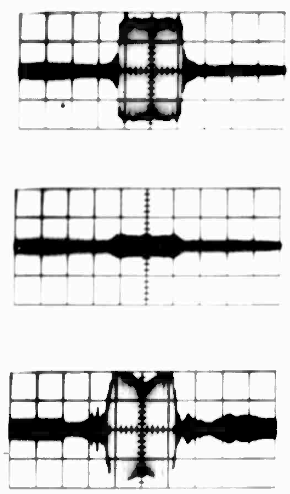


FIGURE 93

TABLE IV (Continued)

Frequency = 10.73 mc | Wedge Angle = 15° | Wedge Material = Lucite | Orientation of cut = up | Mode in Ref. Plate = 5-A

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
I	49	47	49	49	48	45	49	48	48	49	48.1	
A	41	41	41	40	40	41	39	40	42	39	40.4	7.7
I	48	47	43	47	43	45	45	47	46	46	45.7	
B	41	41	39	39	40	40	42	40	39	39	40.0	5.7
I												
C	Very Distorted											
I												
D	Very Distorted											

Figure 94	Figure 95	Figure 96	Figure 97
50 microsec./div.	50 microsec./div.	50 microsec./div.	50 microsec./div.
U: Plate I, 49db	U: Plate I, 46db	U: Plate I, 46db	U: Plate I, 47db
M: Plate A, 49db	M: Plate B, 46db	M: Plate C, 46db	M: Plate D, 47db
L: Plate A, 39db	L: Plate B, 39db	L: Plate C, 30db	L: Plate D, 25db

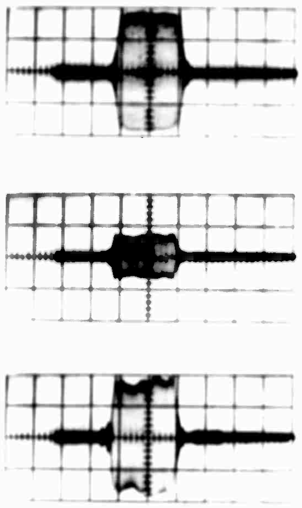


FIGURE 94

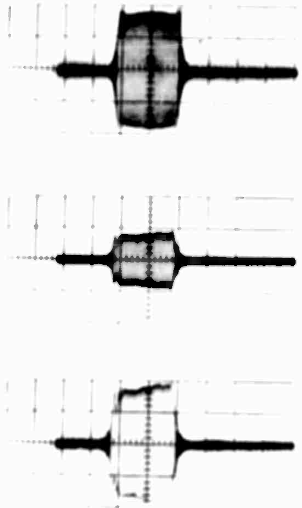


FIGURE 95

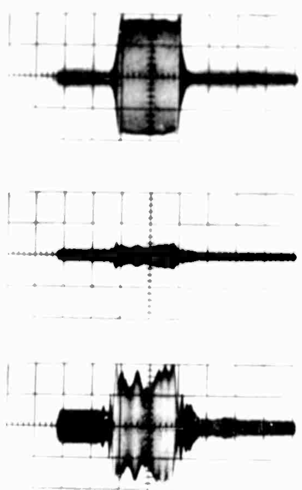


FIGURE 96

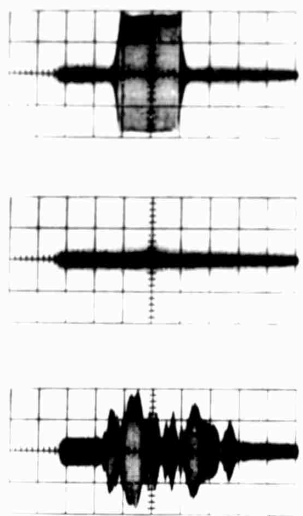


FIGURE 97

TABLE IV (Continued)

Frequency = 10.73 mc					Wedge Angle = 15° Wedge Material = Lucite						Orientation of cut = up Mode in Ref. Plate = 5-A	
Plate	Attenuator Setting, Decibel										Average	Insertion Loss
	Replication No.:											
	1	2	3	4	5	6	7	8	9	10		
I	47	45	47	44	46	42	43	44	45	46	44.9	
E	35	36	36	34	35	35	36	35	36	35	35.4	9.5
I	52	50	49	46	44	48	47	48	49	51	48.4	
F	43	36	39	42	40	39	40	41	40	43	40.3	8.1
I	47	47	48	49	50	50	47	47	48	48	48.1	
G	35	35	33	34	34	30	29	35	36	35	33.6	14.5

Figure 98
50 microsec./div.
U: Plate I, 46db
M: Plate E, 46db
L: Plate E, 46db

Figure 99
50 microsec./div
U: Plate I, 51db
M: Plate F, 51db
L: Plate F, 43db

Figure 100
50 microsec./div.
U: Plate I, 48db
M: Plate G, 48db
L: Plate G, 35db

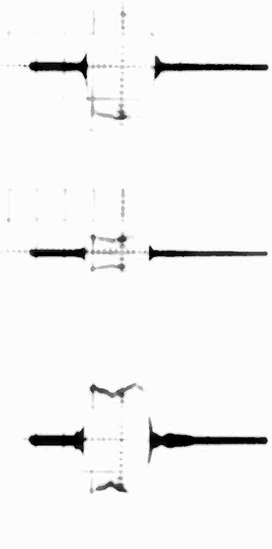


FIGURE 98

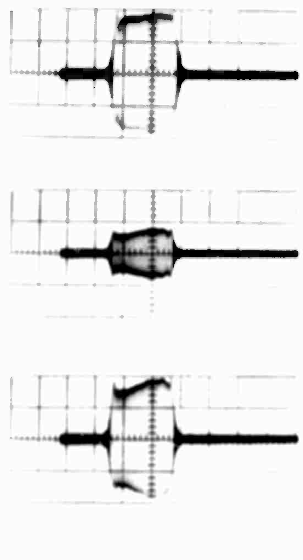


FIGURE 99

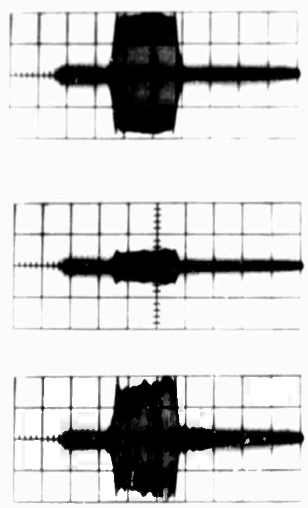


FIGURE 100

The results of Table IV are summarized in Table V.

A complete test was not run with the first asymmetrical mode because the special wedge needed to excite it was difficult to use. However a brief survey indicated that the first asymmetrical mode behaved much as the first symmetrical mode. The second asymmetrical mode was not examined as the combination of trace velocity and frequency-thickness product necessary to excite it was not obtainable with the equipment at hand. For the same reason, no mode above the 5th asymmetrical was excited. Some of the measurements required some judgement in assigning a certain value of amplitude because of the distortion of the pulse. The pictures of each measurement show how much uncertainty was introduced by pulse distortion.

The authors are not able to give a quantitative explanation of the results of these saw cut measurements at this time.

A qualitative explanation is as follows. The energy of the wave incident upon the saw cut is concentrated in a single mode which is determined by the frequency-thickness product of the plate and the transmitting wedge used. When the incident wave reaches the first thickness discontinuity, mode conversion will, in general, take place so that all boundary conditions at the discontinuity will be satisfied. When that part of the wave which is transmitted beneath the saw cut reaches the other side, additional mode conversion must take place so that all three boundaries, two horizontal and one vertical, are satisfied If, as was true in these measurements, the receiving wedge is similar to the transmitting wedge, the receiver will be most sensitive to that mode which was originally transmitted. The energy at the receiving wedge which is effective in producing an output voltage will be approximately equal to the energy which was imparted to the plate from the transmitting wedge less the energy which was reflected back down the plate in various modes less the energy which arrives at the receiving wedge in modes having different phase velocities than the transmitted mode. Since the length of the pulse in the plate is of the order of one or more inches, and since the width of the saw cut is only 1/32", a standing wave is present in the plate below the saw cut; and the width of the saw cut is important in determining its insertion loss.

The very small values of insertion loss measured for the first symmetrical mode in plates A, B, and C can possibly be attributed to the fact that the saw cuts were approximately a half wavelength wide for the first symmetrical mode. The only other mode possible for the region

TABLE V

INSERTION LOSS IN DECIBELS OF VARIOUS DEPTHS OF SAW CUTS

Plate	Depth of Saw Cut	Mode						
		1-S	2-S	3-S	4-S-A	3-A	4-A	5-A
A	0.003"	0.6	6.8	3.9	12.6	5.2	0.6	7.7
B	0.006"	0.1	15.7	4.9	16.1	8.5	1.9	5.7
C	0.009"	-1.9	2.9	-0.6	10.6	5.4	18.4	*
D	0.012"	4.5	12.4	4.0	22.9	23.9	16.9	*
E	0.015"	6.0	4.9	9.9	10.8	10.5	13.1	9.5
F	0.018"	6.9	13.1	6.8	9.2	5.4	*	8.1
G	0.021"	3.8	12.8	5.8	20.7	11.5	*	14.5

* The pulse was too distorted to measure

under the saw cut in this case would be the first asymetrical mode and if very little energy is coupled into this mode, then the insertion loss should be very small.

The higher the order of the mode generated under the transmitting wedge, the more possibilities there are for mode conversion and the more difficult it is to surmise which modes carry the energy beneath the saw cut.

The case where the transmitted pulse appeared to consist of both the fourth symmetrical and fourth asymmetrical modes gave the best indication for all of the plates tested. As mentioned previously, the standard deviation for single observations is about 1 db. The combination fourth symmetrical and assymetrical modes gave an average indication of 9.2 db in the worst case. Consequently, one could expect a minimum indication of 6 db about 99% of the time, and a typical indication as shown in Figure 84.

The fourth and fifth asymmetrical modes showed considerable pulse shape distortion which leads one to think that possibly very high order modes would give the best indication. The indication shown in Figure 89 is nearly ideal.

It is clear from Table V that no simple relationship exists between the depth of the saw cut and the amount of insertion loss. It appears that practical inspection problems may have to be handled on a "go or no go" basis It appears that ultrasonic inspection with Lamb waves is capable of indicating if there is a difference between a given plate and a reference plate. It does not appear that Lamb waves used in this manner will yield very much information about the difference other than the fact that it exists.

3.10.4 Effect of Saw Cut Orientation on Insertion Loss

It was thought worth while to determine the effect of the orientation of the saw cut on the insertion loss. To do this, an unslotted reference plate was inserted in the apparatus and the frequency was adjusted for maximum amplitude. A slotted plate was then placed in the apparatus with its slot up and the amount of attenuation required to make the signal level at the input of the oscilloscope equal to 0.18 volts was recorded. The same plate was then turned over and the attenuation adjustment was again made and recorded. These measurements are tabulated in Table VI and the average values are summarized in Table VII.

In Table VI, the superscript, -1, indicates that the saw cut was down. The only mode where a difference too great to be attributed to experimental error occurred was the composite fourth symmetrical and asymmetrical mode. It is likely that more of the energy is carried on one side of the plate than the other in such a mode due to cancellation of the motion on one side and reinforcement on the other. In some cases the insertion loss was greatest with the saw cut down while in other cases the reverse was true.

TABLE VI

Frequency = 9.4 mc — Wedge Angle = 40° — Wedge Material = Lucite — Mode in Ref. Plate = 2-S

Plate and Orientation of Cut*	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Difference
A	46	48	46	46	45						46.2	
A-1	45	47	46	45	45						45.6	0.6
B	39	39	37	39	35	36	41	39	39	37	38.1	
B-1	35	36	38	38	37	32	37	36	35	39	36.3	1.8
C	44	49	49	49	49	50	46	45	47	47	47.5	
C-1	43	44	48	45	47	45	46	46	46	45	45.5	2.0
D	43	46	47	46	46	47	46	48	44	45	45.8	
D-1	42	46	48	47	46	44	44	46	44	45	45.2	0.6

Figure 101	Figure 102	Figure 103
20 microsec./div.	20 microsec./div.	20 microsec./div.
U: B, 39db	U: C, 47db	U: D, 45db
M: B^{-1}, 39db	M: C^{-1}, 47db	M: D^{-1}, 45db
L: B^{-1}, 35db	L: C^{-1}, 45db	

* A and A^{-1}, for example, indicate that the saw cut was facing up and down respectively.

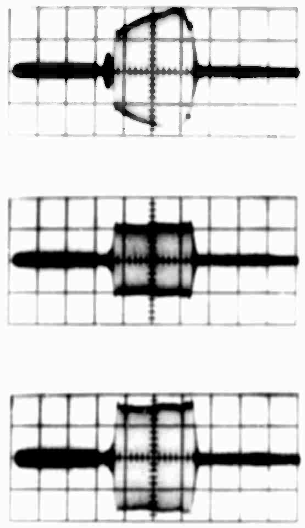


FIGURE 101

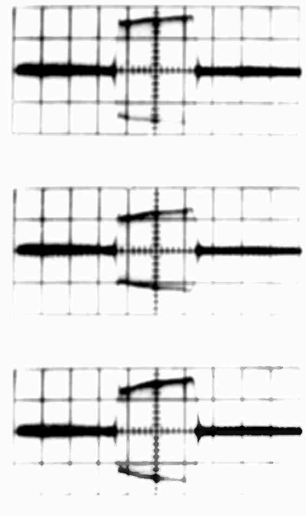


FIGURE 102

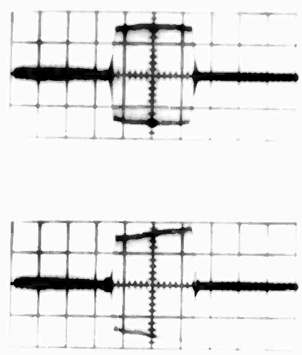


FIGURE 103

TABLE VI (Continued)

Frequency : 9.4 mc — Wedge Angle - 40° — Wedge Material - Lucite — Mode in Ref. Plate - 2-S

Plate and Orientation of Cut	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Difference
E	43	44	42	44	42	44	44	40	42	43	42.8	
E^{-1}	42	44	43	41	43	43	43	42	44	43	42.8	0.0
F	36	38	35	36	35	37	37	37	40	37	36.8	
F^{-1}	37	34	33	36	35	33	34	32	36	35	34.5	2.3
G	39	34	39	35	38	39	39	36	38	39	37.6	
G^{-1}	36	36	37	37	38	35	38	37	35	38	36.7	0.9

Figure 104
20 microsec./div.
U. E, 43db
M: E^{-1}, 43db

Figure 105
20 microsec./div.
U: F, 37db
M: F^{-1}, 37db
L: F^{-1}, 35db

Figure 106
20 microsec./div.
U: G, 39db
M: G^{-1}, 38db

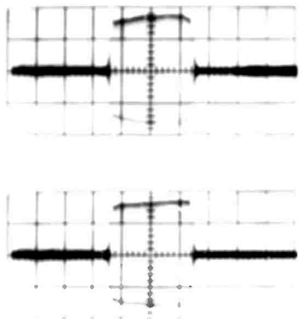


FIGURE 104

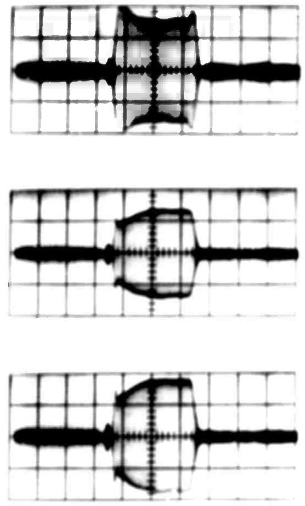


FIGURE 105

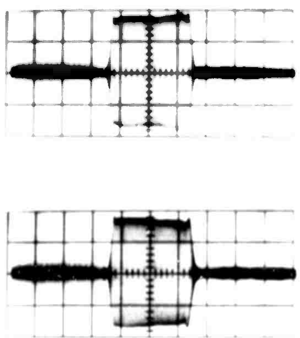


FIGURE 106

TABLE VI (Continued)

Frequency = 10.3 | Wedge Angle = 27.[illegible] | Wedge Material = Lucite | Mode in Ref. Plate = 3-S

Plate and Orientation of Cut	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Difference
A	37	37	39	38	40	39	39	41	39	39	38.8	
A^{-1}	37	36	39	40	38	39	38	40	40	39	38.6	0 2
B	36	35	38	36	34	33	36	36	34	36	35.4	
B^{-1}	38	38	36	37	36	36	36	38	36	35	36.6	-1.2
C	30	30	32	30	31	31	30	30	30	32	30.6	
C^{-1}	33	31	32	29	30	29	29	32	30	32	30.7	-0.1
D	35	33	35	34	34	34	35	34	33	33	34.0	
D^{-1}	35	36	35	35	35	35	35	34	35	34	34.9	-0.9

Figure 107	Figure 108	Figure 109	Figure 110
10 microsec./div.	10 microsec./div.	10 microsec./div.	10 microsec./div.
U. A, 39db	U: B^{-1}, 35db	U: C, 32db	U. D, 33db
M: A^{-1}, 39db	M: B, 36db	M. C^{-1}, 32db	M: D^{-1}, 34db

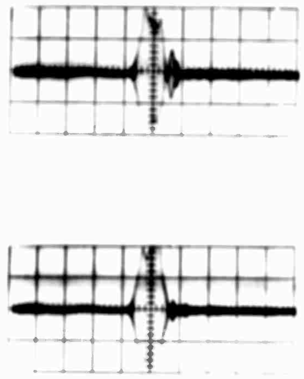


FIGURE 107

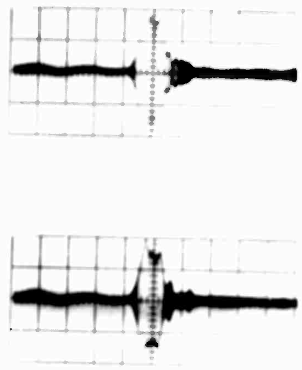


FIGURE 108

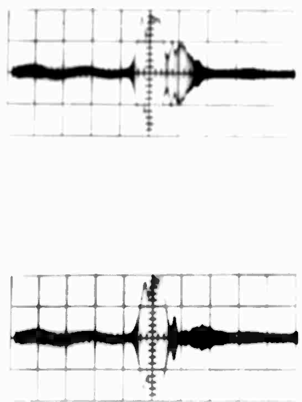


FIGURE 109

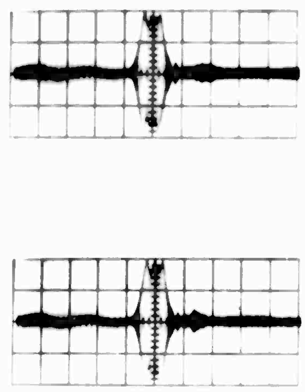


FIGURE 110

TABLE VI (Continued)

Frequency = 10.3 mc				Wedge Angle = 27.5° Wedge Material = Lucite						Mode in Ref. Plate = 3-S	
Plate and Orientation of Cut	Attenuator Setting, Decibels									Average	Difference
	Replication No.										
	1	2	3	4	5	6	7	8	9	10		
E	37	38	37	37	37	37	36	35	37	37	36.8	
E^{-1}	36	36	36	36	36	36	36	35	35	37	35.9	0.9
F	34	34	34	32	34	35	34	34	34	33	33.8	
F^{-1}	32	32	32	31	34	32	33	33	33	33	32.5	1.3
G	30	30	30	30	30	29	30	29	29	30	29.7	
G^{-1}	31	29	31	29	30	30	30	30	30	31	30.1	-0.4

Figure 111
10 microsec./div.
U. E, 37db
M. E^{-1}, 37db

Figure 112
10 microsec./div.
U: F, 33db
M. F^{-1}, 33db

Figure 113
10 microsec./div.
U. G, 30db
M: G^{-1}, 31db

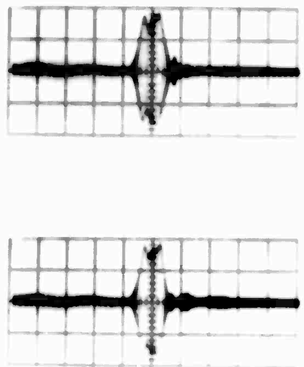


FIGURE 111

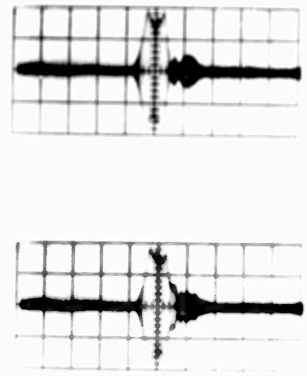


FIGURE 112

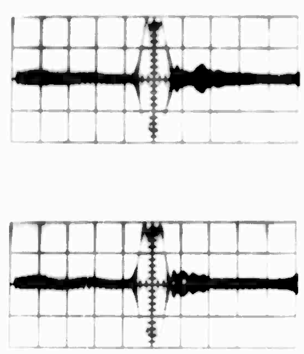


FIGURE 113

TABLE VI (Continued)

Frequency = 9.3 mc | Wedge Angle = 30° | Wedge Material = Lucite | Mode in Ref. Plate = 3-A

Plate and Orientation of Cut	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Difference
A	39	40	40	42	43	43	42	40	39	39	40.7	
A^{-1}	39	40	41	42	43	41	40	37	39	38	40.0	0.7
B	36	39	39	40	39	39	40	39	38	40	38.9	
B^{-1}	37	38	39	40	38	39	37	38	39	39	38.4	0.5
C	37	40	38	40	38	39	39	40	40	41	39.2	
C^{-1}	40	41	41	42	41	40	39	41	42	42	40.9	-1.7
D	31	30	29	31	29	27	27	29	28	28	28.9	
D^{-1}	30	28	28	28	28	28	27	28	29	28	28.2	0.7

Figure 114	Figure 115	Figure 116	Figure 117
20 microsec./div.	20 microsec./div.	20 microsec./div.	20 microsec./div.
U: A, 39db	U. B, 40db	U: C, 41db	U: D, 28db
M: A^{-1}, 39db	M: B^{-1}, 39db	M: C^{-1}, 42db	M: D^{-1}, 28db

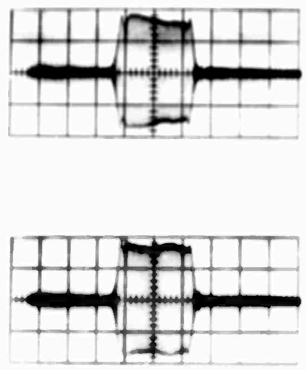


FIGURE 114

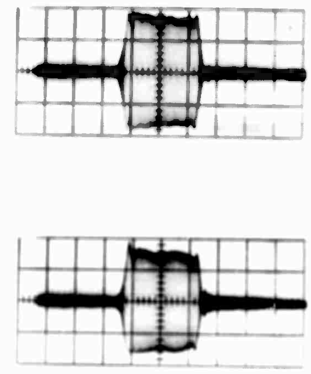


FIGURE 115

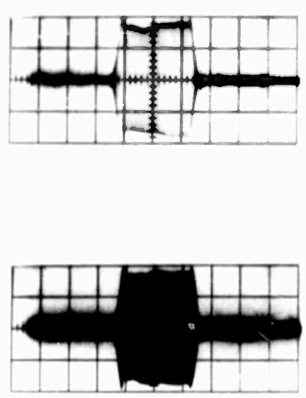


FIGURE 116

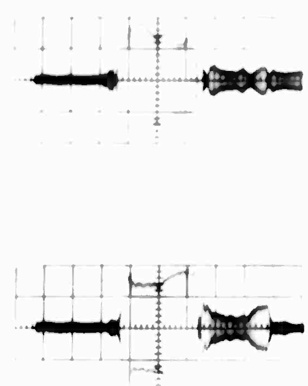


FIGURE 117

TABLE VI (Continued)

Frequency = 9.3 mc — Wedge Angle = 30° — Wedge Material = Lucite — Mode in Ref. Plate = 3-A

Plate and Orientation of Cut	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Difference
E	34	35	33	36	37	36	35	35	36	35	35.2	
E^{-1}	34	35	37	36	37	36	36	36	36	36	35.9	-0.7
F	37	36	38	36	38	37	34	33	33	35	35.7	
F^{-1}	35	35	37	37	34	33	31	33	31	35	34.1	1.6
G	36	36	35	38	37	37	36	36	36	36	36.3	
G^{-1}	36	36	34	34	34	35	35	36	36	36	35.2	1.1

Figure 118	Figure 119	Figure 120
U: C, 41db	U: D, 28db	U: E, 35db
M: C^{-1}, 42db	M: D^{-1}, 28db	M: E^{-1}, 36db

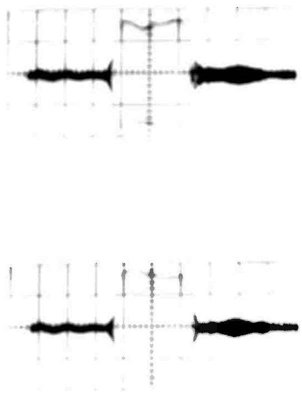


FIGURE 118

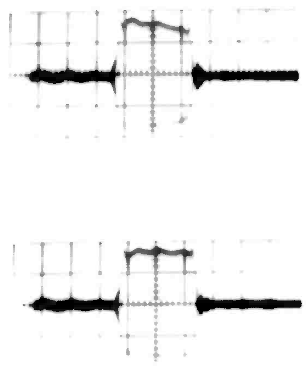


FIGURE 119

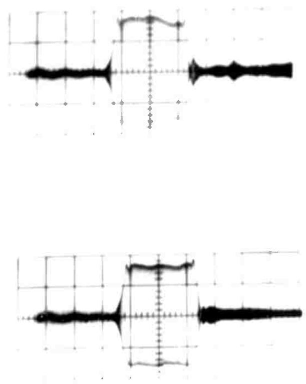


FIGURE 120

TABLE VI (Continued)

Frequency = 9.15 mc | Wedge Angle = 20° | Wedge Material = Lucite | Mode in Ref. Plate = 4-S-A

Plate and Orientation of Cut	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Difference
A	43	42	42	42	43	42	43	43	42	40	42.2	
A^{-1}	37	37	36	42	32	35	35	35	36	34	35.9	6.3
B	44	43	46	43	44	45	43	44	44	46	44.2	
B^{-1}	43	35	41	38	42	42	38	40	41	42	40.2	4.0
C	35	37	36	41	40	40	39	39	36	34	37.7	
C^{-1}	44	43	43	42	43	44	44	46	43	44	43.6	-5.9
D	31	27	34	32	30	34	28	28	30	32	30.6	
D^{-1}	44	44	46	46	43	45	40	47	45	44	44.4	-13.8

Figure 121	Figure 122	Figure 123	Figure 124
20 microsec./div.	20 microsec./div.	20 microsec./div.	20 microsec./div.
U: A, 40db	U: B, 46db	U: C^{-1}, 43db	U: D^{-1}, 44db
M: A^{-1}, 40db	M: B^{-1}, 46db	M: C, 43db	M: D, 44db
L: A^{-1}, 34db	L: B^{-1}, 42db	L: C, 38db	L: D, 32db

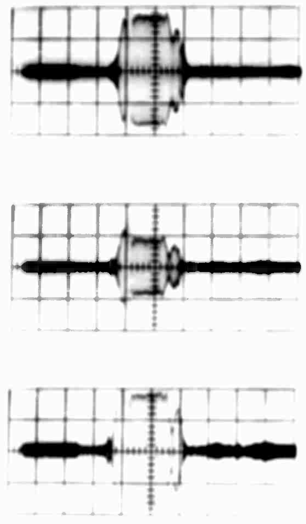


FIGURE 121

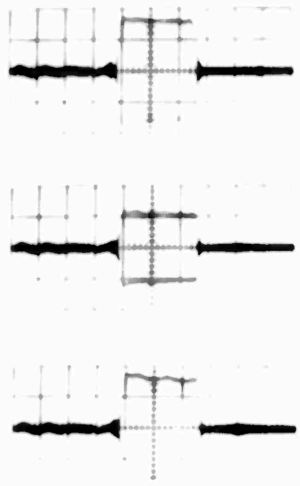


FIGURE 122

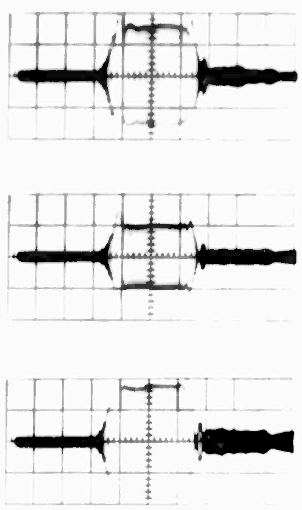


FIGURE 123

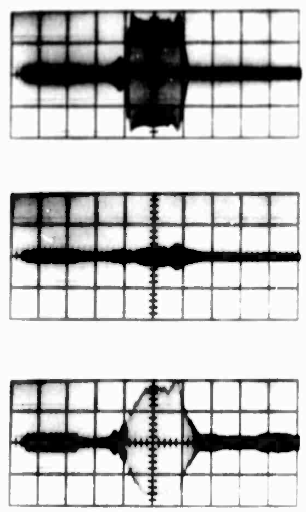


FIGURE 124

TABLE VI (Continued)

Frequency = 9.15 mc — Wedge Angle = 20°, Wedge Material = Lucite — Mode in Ref. Plate = 4-S-A

Plate and Orientation of Cut	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Difference
E	33	37	40	35	39	37	34	35	39	35	36.4	
E^{-1}	33	36	32	35	38	33	39	36	37	34	35.3	1.1
F	38	39	44	43	41	38	39	38	38	38	39.6	
F^{-1}	32	32	34	31	33	30	33	30	31	32	31.8	7.8
G	40	35	40	36	43	33	39	38	32	41	37.7	
G^{-1}	42	44	41	41	38	42	44	38	36	37	40.3	-2.6

Figure 125
20 microsec./div.
U: E, 35db
M: E^{-1}, 34db

Figure 126
20 microsec./div.
U. F, 38db
M: F^{-1}, 38db
L: F^{-1}, 32db

Figure 127
20 microsec./div.
U: H, 50db
M: G^{-1}, 37db
L: G, 41db

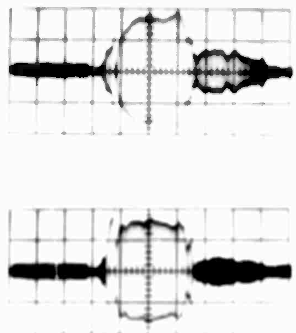


FIGURE 125

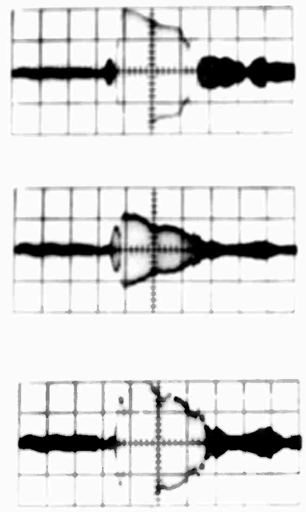


FIGURE 126

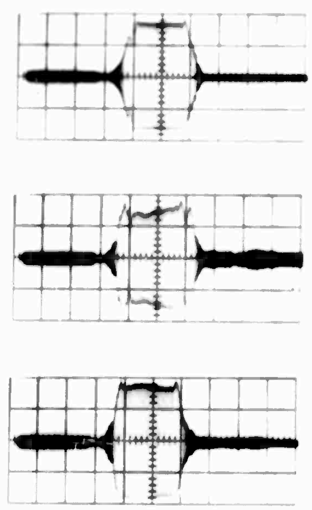


FIGURE 127

TABLE VII

Effect of the Orientation of Test Plates Upon the Transmission Loss of Ultrasound-The Transmission Loss with the Saw Cut Down Referred to the Transmission Loss with the Saw Cut Up-In Decibels

Plate \ Mode	2-S	3-S	3-A	4-S-A
A	0.6	0.2	0.7	6.3
B	1.8	-1.2	0.5	4.0
C	2.0	-0.1	-1.7	-5.9
D	0.6	-0.9	0.7	-13.8
E	0.0	0.9	-0.7	1.1
F	2.3	1.3	1.6	7.8
G	0.9	-0.4	1.1	-2.6

In the case of plate C, the insertion loss of the composite fourth symmetrical and asymmetrical mode was only 4.7 db when the plate was oriented with the slot down. If 3db is subtracted from this to allow for experimental error, the minimum indication is only 1.7 db which probably would not be enough in a practical testing situation. However, the other saw cuts produced enough insertion loss for this mode so that an operator should be able to distinguish them from the uncut plates on the basis of the amount of insertion loss.

While these measurements were being made, it was noticed that the frequency at which a given mode was found differed slightly from the frequency of which it was found in the measurements of section 3.10.3. This was attributed to not being able to adjust the angles of the positioning clamp exactly the same in both sets of measurements.

3.11 Experiment on Welded Plates

Nine 2" × 10" × 1/32" plates were cut from one 10" × 18" × 1/32" piece of precision ground flat stock, Starrett type No. 496. Eight of these plates were cut in two at the center and the two halves were butt welded together. An attempt was made to vary the quality of the welds by varying the amount of heat used. The plates were all annealed, including plate J which was not cut and welded, and then the weld beads were dressed down. Plates J and N are shown in Figure 128. The insertion loss of each of these welds was measured following the procedure of section 3.10.3, and the data obtained are given in Table VIII. The values of insertion loss for each welded plate at the modes used are summarized in Table IX

These plates were destructively tested in a testing machine. In order to insure that the plates would break near the middle rather than near a jaw of the testing machine, cuts were made in each plate as shown in Figure 129. The results of the destructive tests are given in Table X.

The worst of these plates failed at a stress of only 17% less than plate J. The strongest weld was in plate K; however, the insertion loss of this weld was rather large for both the third symmetrical and the composite fourth symmetrical and asymmetrical mode. Plate R was the weakest plate and its largest value of insertion loss was 6.3 db for the third symmetrical mode. However, the weakness of this plate was probably due to the hole in it. Plates M, N and Q had measured insertion losses of 15 db or more for one or more modes, so that the correlation between weld strength and insertion loss does not seem to be particularly good.

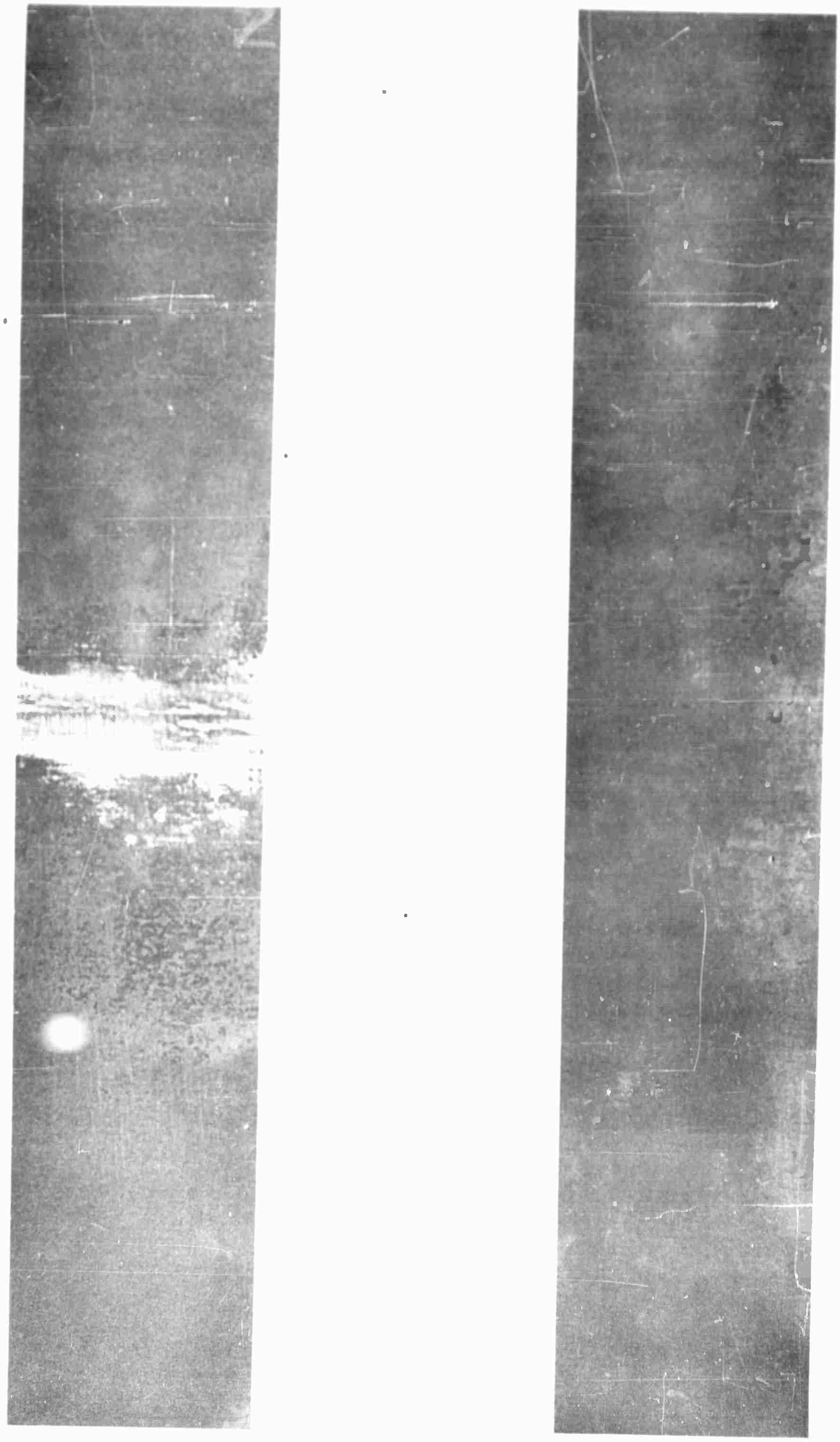


FIGURE 128

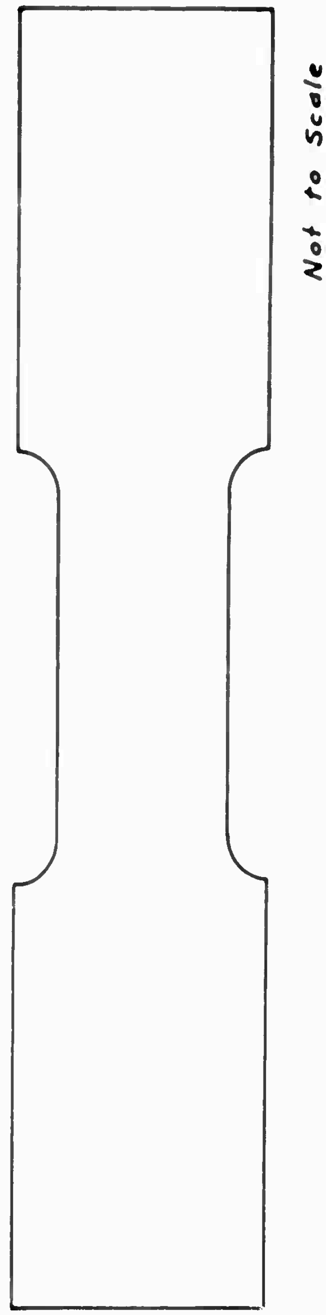


FIGURE 129

TABLE VIII

Frequency = 9.07 mc   Wedge Angle = 40°   Wedge Material = Lucite   Mode in Ref. Plate = 2-S

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
J	36	36	34	33	36	34	34	33	36	35	34.7	
K	31	30	28	30	28	29	29	32	31	31	29.9	4.8
J	40	38	39	34	37	35	36	38	36	38	37.1	
L	43	41	39	39	44	45	44	43	46	46	43.0	-5.9
J	35	36	36	37	39	39	38	38	36	37	37.1	
M	35	37	36	38	38	38	38	37	37	38	37.2	-0.1
J	34	39	35	38	39	39	37	39	39	39	37.8	
N	35	34	33	33	33	32	34	33	33	33	33.3	4.5

Figure 130	Figure 131	Figure 132	Figure 133
10 microsec./div.	20 microsec./div.	20 microsec./div.	20 microsec./div.
U: J, 36db	U: L, 46db	U: J, 37db	U: J, 39db
M: K, 36db	M: J, 46db	M: M, 38db	M. N, 39db
L: K, 31db	L: J, 38db		L: N, 33db

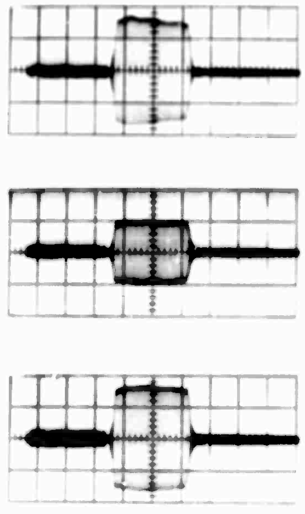


FIGURE 130

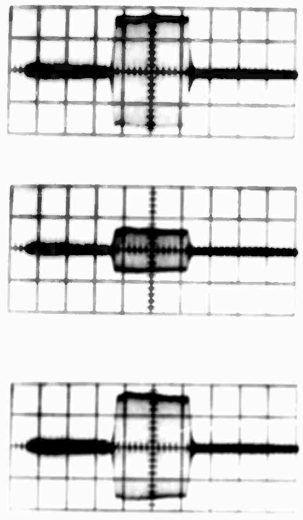


FIGURE 131

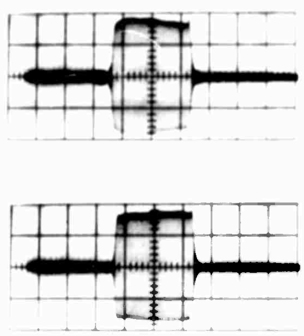


FIGURE 132

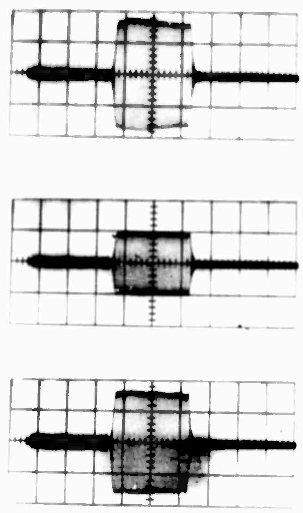


FIGURE 133

TABLE VIII (Continued)

Frequency = 9.07 mc | Wedge Angle = 40° | Wedge Material = Lucite | Mode in Ref. Plate = 2-S

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
J	41	37	39	37	41	35	39	40	40	39	38.8	
O	35	36	35	37	33	34	39	34	35	35	35.3	3.5
J	39	42	38	39	41	42	38	40	40	40	39.9	
P	29	37	36	33	33	35	36	31	37	30	33.7	6.2
J	39	41	40	39	40	40	39	35	38	38	38.9	
Q	33	31	36	32	32	35	32	30	30	30	32.1	6.8
J	40	40	41	39	39	38	37	40	36	38	38.8	
R	45	45	43	42	44	45	46	44	46	44	44.4	-5.6

Figure 134	Figure 135	Figure 136	Figure 137
20 microsec./div.	20 microsec./div.	20 microsec./div.	20 microsec./div.
U: J, 39db	U: J, 40db	U: J, 38db	U: R, 44db
M: 0, 39db	M: P, 40db	M: Q, 38db	M: J, 44db
L: 0, 35db	L: P, 30db	L: Q, 30db	L: J, 38db

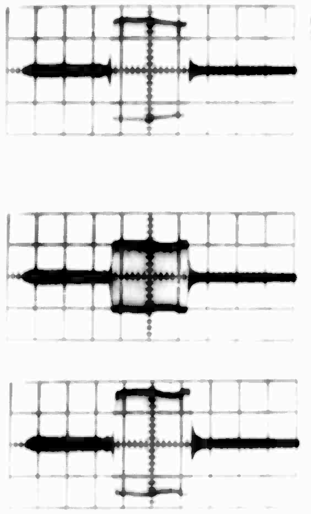


FIGURE 134

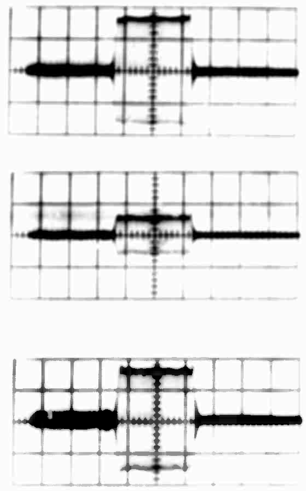


FIGURE 135

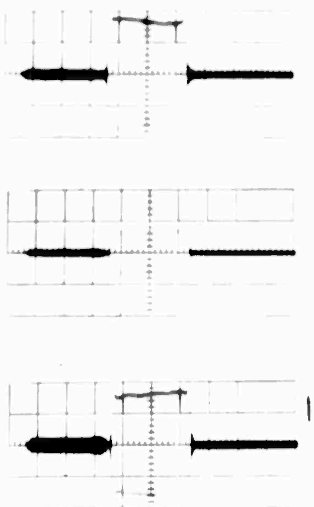


FIGURE 136

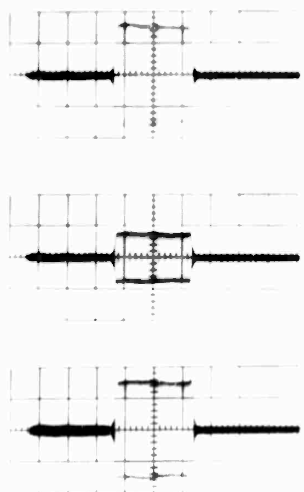


FIGURE 137

TABLE VIII (Continued)

Frequency = 9.93 mc

Wedge Angle = 30°
Wedge Material = Lucite

Mode in Ref. Plate = 3-S

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
J	37	40	38	36	38	39	40	37	40	37	38.2	
K	25	26	23	30	27	24	27	26	30	25	26.3	11.9
J	38	38	37	38	37	38	37	34	36	37	37.0	
L	36	36	39	38	40	37	39	42	35	35	37.7	-0.7
J	40	37	41	38	35	41	40	36	39	35	38.2	
M	24	25	19	24	27	23	19	19	20	23	22.3	15.9
J	38	31	37	37	38	36	37	36	38	38	36.6	
N	19	19	19	22	19	21	18	23	18	26	20.4	16.2

Figure 138	Figure 139	Figure 140	Figure 141
10 microsec./div.	10 microsec./div.	10 microsec./div.	10 microsec./div.
U: J, 37db	U: J, 37db	U: J, 35db	U: J, 38db
M: K, 37db	M: L, 37db	M: M, 35db	M: N, 38db
L: K, 25db	L: L, 35db	L: M, 23db	L: N, 26db

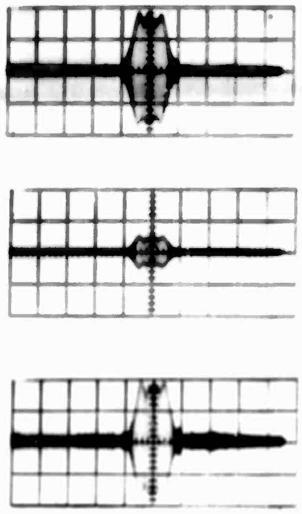


FIGURE 138

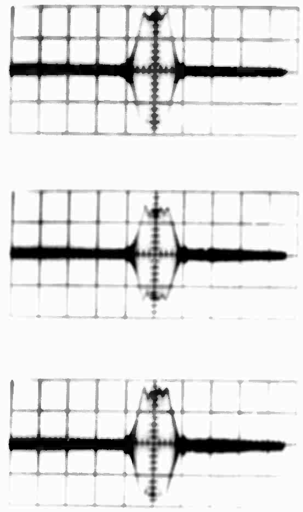


FIGURE 139

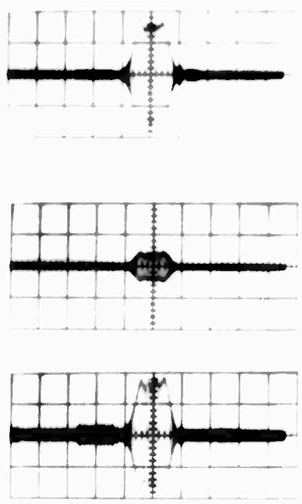


FIGURE 140

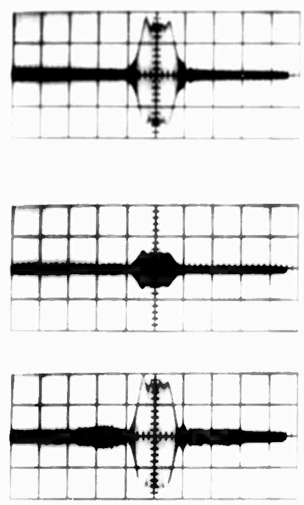


FIGURE 141

TABLE VIII (Continued)

Frequency = 9.93 mc					Wedge Angle = 30° Wedge Material = Lucite						Mode in Ref. Plate = 3-S	
Plate	Attenuator Setting, Decibels										Average	Insertion Loss
	Replication No.:											
	1	2	3	4	5	6	7	8	9	10		
J	36	36	39	41	41	41	40	35	36	41	38.6	
O	25	24	25	32	31	28	30	25	26	29	27.5	11.1
J	41	37	34	38	37	35	38	36	38	37	37.1	
P	33	33	30	32	33	30	32	33	32	33	32.1	5.0
J	38	40	36	34	41	37	38	37	35	39	37.5	
Q	20	22	22	28	26	26	20	22	19	20	22.5	15.0
J	41	37	36	38	36	40	39	37	36	39	37.9	
R	32	34	31	29	34	29	32	33	34	28	31.6	6.3

Figure 142	Figure 143	Figure 144	Figure 145
10 microsec./div.	10 microsec./div.	10 microsec./div.	10 microsec./div.
U: J, 41db	U: J, 37db	U: J, 39db	U: J, 39db
M: 0, 41db	M: P, 37db	M: Q, 39db	M: R, 39db
L: 0, 29db	L: P, 33db	L: Q, 20db	L: R, 28db

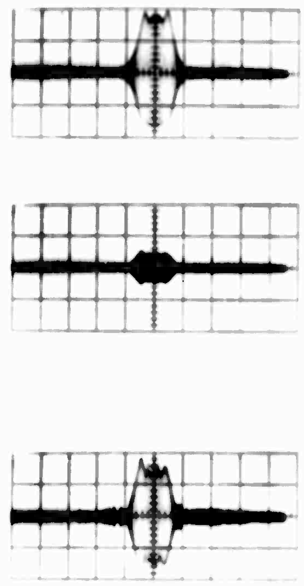


FIGURE 142

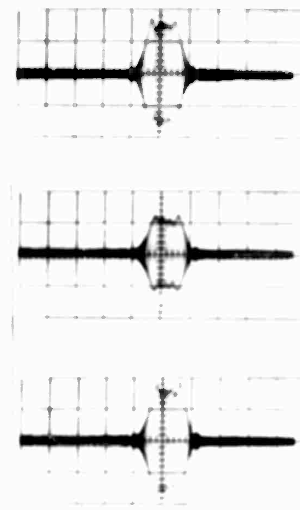


FIGURE 143

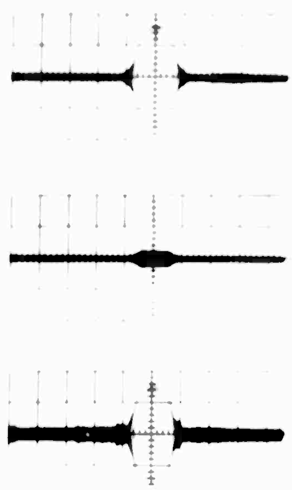


FIGURE 144

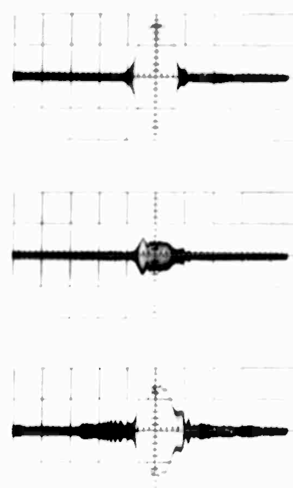


FIGURE 145

TABLE VIII (Continued)

Frequency = 9.15 mc — Wedge Angle = 20° — Wedge Material = Lucite — Mode in Ref. Plate = 4-S-A

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
J	52	46	45	47	49	48	49	51	51	48	48.6	
K	37	36	37	38	37	36	38	38	40	38	37.5	11.1
J	47	49	49	50	49	49	49	50	50	49	49.1	
L	52	49	48	51	48	50	45	48.	47	49	48.7	0.4
J	49	49	44	46	47	50	49	49	48	50	48.1	
M	33	27	30	32	32	31	31	31	30	32	30.9	17.2
J	48	50	49	49	48	49	49	47	48	49	48.6	
N	34	34	37	34	32	36	36	34	33	36	34.6	14.0

Figure 146	Figure 147	Figure 148	Figure 149
20 microsec./div.	20 microsec./div.	20 microsec./div.	20 microsec./div.
U: J, 48db	U: J, 49db	U: J, 50db	U: J, 49db
M: K, 48db	M: L, 49db	M: M, 50db	M: N, 49db
L: K, 38db		L: M, 32db	L: N, 36db

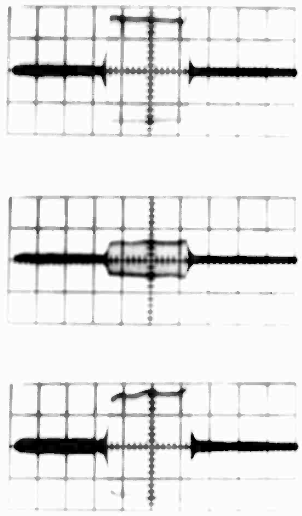


FIGURE 146

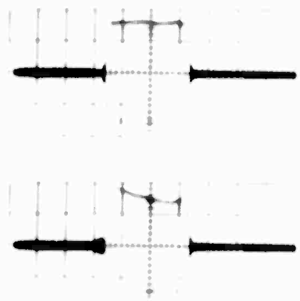


FIGURE 147

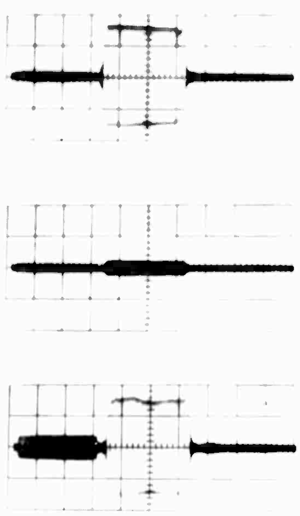


FIGURE 148

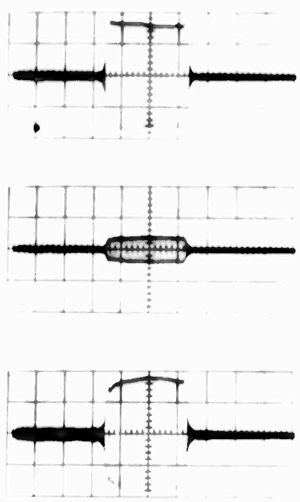


FIGURE 149

TABLE VIII (Continued)

Frequency = 9.15 mc. Wedge Angle = 20° Wedge Material = Lucite Mode in Ref. Plate = 4-S-A

Plate	Attenuator Setting, Decibels — Replication No.: 1	2	3	4	5	6	7	8	9	10	Average	Insertion Loss
J	49	48	47	47	48	47	47	47	47	49	47.6	
O	38	36	35	39	36	39	37	40	39	41	38.0	9.6
J	49	49	47	49	48	47	47	45	49	47	47.7	
P	39	45	45	41	40	42	45	44	44	41	42.6	5.1
J	49	49	49	49	49	49	46	47	47	45	47.9	
Q	41	45	43	46	43	43	41	41	45	40	42.8	5.1
J	47	47	45	46	46	46	46	44	46	46	45.9	
R	47	44	47	49	46	48	46	45	45	46	46.3	-0.4

Figure 150	Figure 151	Figure 152	Figure 153
20 microsec./div.	20 microsec./div.	20 microsec./div.	20 microsec./div.
U: J,49db	U: J,47db	U: J,45db	U: J,46db
M. 0,49db	M: P,47db	M: Q,45db	M. R,46db
L: 0,41db	L: P,41db	L: Q,40db	

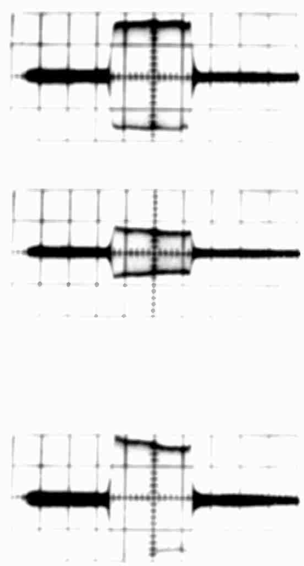


FIGURE 150

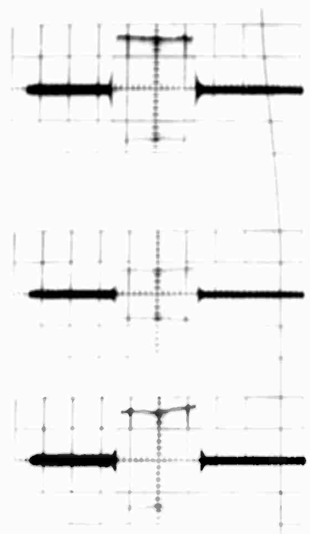


FIGURE 151

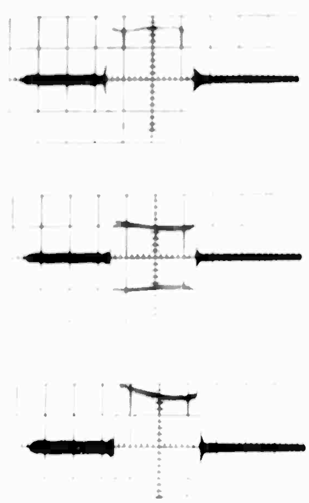


FIGURE 152

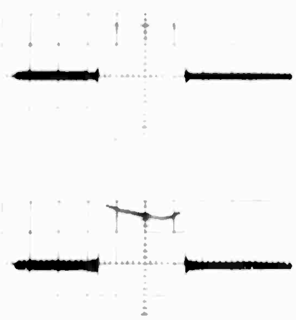


FIGURE 153

TABLE IX

INSERTION LOSS OF BUTT WELDS IN DECIBELS

Plate \ Mode	2-S	3-S	4-S-A
K	4.8	11.9	11.1
L	-5.9	-0.7	0.4
M	-0.1	15.9	17.2
N	4.5	16.2	14.0
O	3.5	11.1	9.6
P	6.2	5.0	5.1
Q	6.8	15.0	5.1
R	-5.6	6.3	-0.4

None of these welds was significantly weaker than the unwelded plate. However, the insertion loss of most of them was great enough at one or more modes to lead one to think them defective.

It is quite possible that if a fatigue test were used in the destructive testing of these plates, the correlation between insertion loss and the destructive test results would be better. The fatigue test would also be a better criterion of the soundness of the weld.

3.12 Detection of Flaws by "Scattering"

Gericke has reported a method of detecting flaws very close to the surface[1] which is illustrated in Figure 154. The transmitter sends ultrasound down the test piece, and a flaw such as is shown scatters it, so that the receiver picks up a signal when it is over the flaw. However, as the receiver is moved away from the flaw the amplitude of the received pulse decreases rapidly. This suggests the use of Lamb waves when the test piece is a thin plate, and the experiment illustrated in Figure 155 was performed. The 30 lucite wedge was used to generate the third asymmetrical mode at 10.6 mc. An exposure was made for each of six positions spaced 3/16" apart in the vicinity of the saw cut, and this procedure was repeated for each of the test plates described in section 3.10.2. These photographs are Figures 156-162. Small tuning adjustments were made prior to taking each of these photographs, to maximize the signal over each saw cut to demonstrate the capabilities of the method. There are six exposures in each photograph. The quantity, x, is the distance between the center of the transducer and the center of the saw cut which is taken to be positive when the transducer is to the right of the saw cut. The attenuator setting and frequency are given with each photograph as well as the time scale. The transducers were clamped for these photographs. When the receiving transducer was positioned manually, the deeper cuts could be located in this way, but an attempt to locate the 0.003" cut was not successful.

It is not quite correct to say that the pulse is scattered in the case of a thin plate since the energy is still confined to the plate. A better explanation, in this case, is that a standing wave is set up in the metal above the saw cut which results in greater particle displacement than in the rest of the plate.

---

[1] Gericke, O.R. Ultrasonic Methods for Near-Surface Flaw Detection, Watertown Arsenal, Watertown, Mass., 1959, AD 229387.

TABLE X

DESTRUCTIVE TESTS OF WELDED PLATES

Plate	Width, in.	Thickness in.	Cross Sectional Area, in.^{2}	Ultimate Load, lbs	Ultimate Strength, Psi	Comments on Failure
J	1.499	0.033	0.0495	5,050	102,000	
K	1.500	0.032	0.0480	5,005	104,000	started in weld
L	1.501	0.032	0.0481	4,570	95,000	
M	1.501	0.031	0.0466	4,210	90,400	started in hole
N	1.500	0.032	0.0480	4,590	95,600	failed outside of weld
O	1.501	0.032	0.0481	4,390	91,400	
P	1.501	0.032	0.0481	4,610	95,900	started in weld
Q	1.501	0.032	0.0481	4,410	91,600	started in hole
R	1.500	0.032	0.0480	4,060	84,600	started in hole

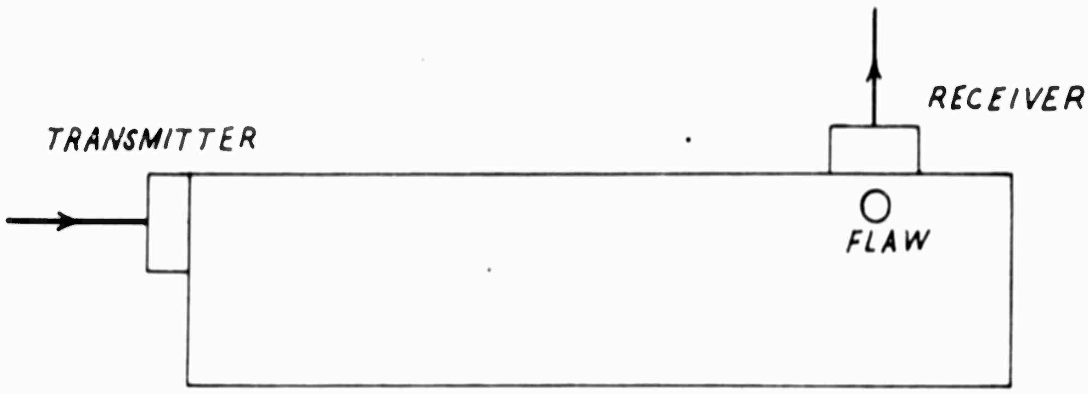


FIGURE 154

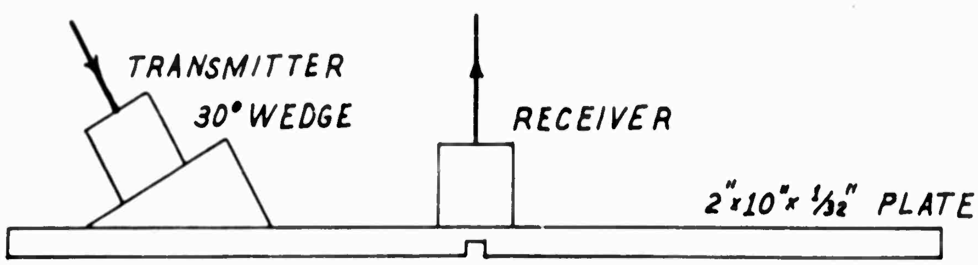
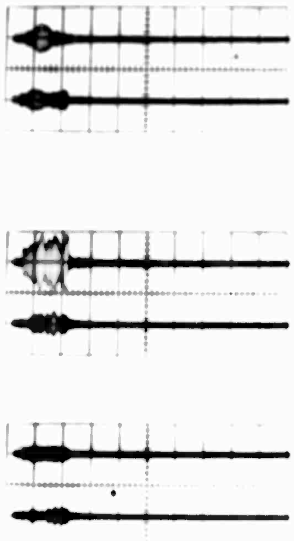


FIGURE 155



x = -3/8"

x = -1/4"

x = -1/8"

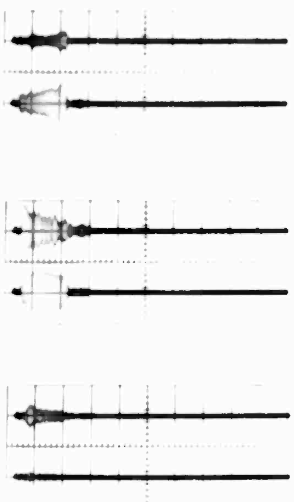
x = 0

x = +1/8"

x = +1/4"

Plate A
52 db, 10.60 mc
100 microsec./div.

FIGURE 156



x = -3/8"

x = -1/4"

x = -1/8"

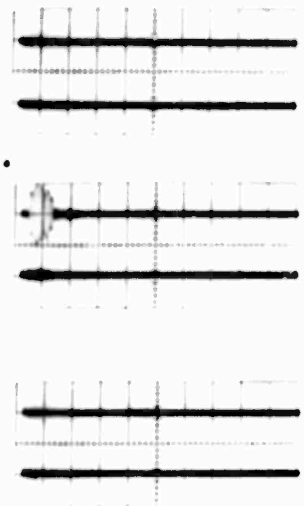
x = 0

x = +1/8"

x = +1/4"

Plate B
49 db, 10.65 mc
100 microsec./div.

FIGURE 157



x = -3/8"

x = -1/4"

x = -1/8"

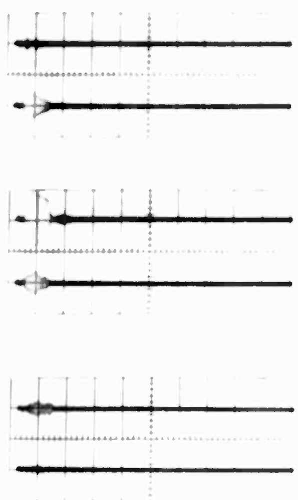
x = 0

x = +1/8"

x = +1/4"

Plate C
61db, 10.03 mc
100 microsec./div.

FIGURE 158



x = -3/8"

x = -1/4"

x = -1/8"

x = 0

x = +1/8"

x = +1/4"

Plate D
56 db, 10.60 mc
100 microsec./div.

FIGURE 159

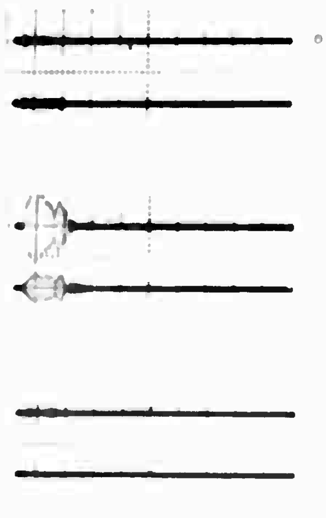


Plate E
50 db, 10.53 mc
100 microsec./div.

FIGURE 160

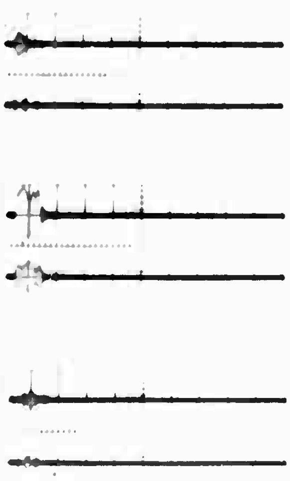


Plate F
46 db, 10.60 mc.
100 microsec./div.

FIGURE 161

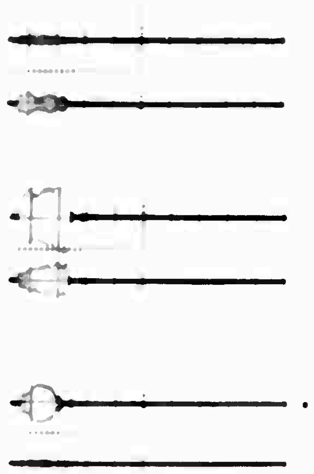


Plate G
53 db, 10.7 mc
100 microsec. div.

FIGURE 162

SUMMARY AND DISCUSSION OF FUTURE WORK

4.1 Summary

A bibliography has been compiled which contains one hundred and six abstracts of publications concerned with the propagation of ultrasound within plates and with related subjects. This bibliography is given at the end of the report.

A survey of the theory of vibrations of thin elastic plates has been given. This survey contains what was thought to be the most important aspects of the theory, although it is evident from the bibliography that it does not cover all of the theory which has been developed. In fact no pretense is made that all of the theory and experimental work represented by the bibliography could possibly be assimilated in the amount of time which was available.

The period equation giving phase velocity as a function of frequency-thickness product was normalized and the symmetrical modes for a ratio of longitudinal velocity to shear velocity of 1.8 were computed using the Datatron 205 digital computer. The corresponding values of group velocity were also computed. Curves were plotted of both phase velocity and group velocity versus frequency-thickness product for this case.

The excitation of vibrations in infinite plates by the use of sine wave pulses is discussed with reference to the spectra of sine wave pulses. A formula is derived for computing group velocity from phase velocity and the derivative of phase velocity with respect to frequency-thickness product.

Some of the experimental work described was undertaken to gain confidence in the techniques used and to contrast the behavior of thick and thin plates. Consequently, some of the experiments described were performed on thick plates and it is shown that these plates were not vibrating in the modes properly called Lamb waves.

It was found that the apparatus used (very similar to the type of apparatus usually used in practical nondestructive testing procedures) produced Lamb waves in plates which were of the order of one wavelength thick. Plates which were of the order of ten wavelengths thick did not vibrate in these modes and plates which were of intermediate thickness may have simultaneously contained multiply-reflected shear and longitudinal waves as well as Lamb waves.

In the process of studying thick plates by way of comparison, it was found necessary to consider the effect of beam divergence in explaining the waveforms which were observed.

Measurements were made of the group velocity of Lamb waves, and the experimental and calculated values were found to agree satisfactorily.

Some modes of vibration were observed in 1/8" and 1/4" thick plates which can not be explained either as Lamb waves or multiply reflected shear or longitudinal waves.

Measurements were made of the insertion loss of saw cuts in 1/32" thick steel plates. These saw cuts were 0.003", 0.006", 0.009", 0.012", 0.015", 0.018", and 0.021" deep. No simple relationship was discovered between the depth of the saw cut and the amount of insertion loss; and, in fact, one is not justified in expecting any simple relationship since mode conversion will take place at a thickness discontinuity. Measurgments were also made to determine whether the insertion loss of a saw cut is different when the saw cut is on the same side of the plate as the transducers than when the saw cut is on the opposite side of the plate. (In all of these experiments both transducers were on the same side of the plate.) It was found that there was no significant difference except for the case where the fourth symmetrical and asymmetrical modes were being generated simultaneously. When power is propagated through a plate in a composite symmetrical and asymmetrical mode of this type, there will be some cancellation of particle displacement on one side of the plate and reinforcement of particle displacement on the other side. Consequently, more power will be propagated along one side of the plate than along the other so that one would expect a flaw to have more effect on one side than on the other.

An experiment was performed in an attempt to determine the effectiveness of Lamb waves for testing welds. In several cases, an insertion loss of 15 db or more was measured. However, when the welds were destructively tested by static loading, none of them could be classified as weak.

A method of locating flaws in thin plates was investigated which consisted of using a lucite wedge for generating a Lamb wave in the plate and searching for the flaw with the receiving transducer held directly in contact with the plate. This method shows promise.

This study has yielded knowledge of some of the properties of Lamb waves which have importance in their application to nondestructive testing. The question of whether or not Lamb waves will find extensive application in the testing of thin plates will have to await attempts to use them in specific cases. It is felt that the knowledge of Lamb wave phenomena contained in this report should be useful in developing such applications.

4.2 Discussion of Future Work

It is earnestly hoped that funds can be made available to permit the continuation of this work. Some of the areas which the authors hope to investigate are as follows:

4.2.1 Investigation of the Insertion Loss of Saw Cuts at Higher Order Modes

When measurements were made of the insertion loss of the fourth asymmetrical mode, the test plates F and G produced extreme distortion of the pulse (Figures 85 and 86) which was accompanied by considerable attenuation. The same effect was noticed in the case of the fifth symmetrical mode and plates C and D (Figures 96 and 97). This leads one to think that there might be even more pulse distortion caused by flaws at higher order modes. Furthermore this seems reasonable, because the higher the order of the original mode, the more possibilities there are for mode conversion. In order to explore higher order modes, it will be necessary to drive the 10 mc transducers at their next resonant frequency or 30 mc. In order to work in the neighborhood of 30 mc., some electronic design work may be necessary to resonate the electrical capacity of the transducers with inductance and to construct a special low noise preamplifier since the signal level will undoubtedly be much lower at 30 mc. than at 10 mc.

4.2.2 Study of the Reflection, Transmission, and Mode Conversion of Lamb Waves in Plates Where There is a Change of Thickness

In analyzing the data obtained in the experiment on the plates with saw cuts, it became evident that a clearer understanding of the nature of Lamb wave reflection, transmission, and mode conversion would be obtained if experiments were performed on plates which had one thickness for half of their length and a different thickness for the other half. Such plates would be obtained by starting with a set of

1/32" plates and machining half of each one down to a different thickness as shown in Figure 163. When a Lamb wave is incident upon the thickness discontinuity, mode conversion will take place, and there will, in general, be several reflected modes and several transmitted modes. The objective of this experiment will be to attempt to measure the various reflection and transmission coefficients involved and to identify the reflected and transmitted modes. It would be desirable to repeat this experiment using plates which have a gradual transition from one thickness to the other as shown in Figure 164. In parallel with this experimental work, a theoretical investigation would be carried out to try to analyze as many special cases as possible and to work towards a method for analysis of the general case.

4.2.3 Further Investigation of Weld Testing

The experiment on welded plates described in this report was inconclusive because, in several cases, an insertion loss of 15 db or more was measured even though none of the welds could be considered weak when destructively tested by static loading. In order to better evaluate the use of Lamb waves for testing welds in thin plates, another experiment should be performed in which a greater variation in welding heat will be used when the plates are welded, and an attempt will be made to correlate the ultrasonic indications with a fatigue test. The fatigue test is a better criterion of weld quality than static loading.

4.2.4 Surface Roughness

The experiments performed so far have been on plates which had smooth surfaces. In future work, an experiment will be performed to determine the effect of surface roughness upon the propagation of Lamb waves.

4.2.5 "Scattering" Technique

Since the "scattering" method of searching for flaws with Lamb waves shows promise, further development of this technique is planned.

4.2.6 Love Waves

It is felt that the scope of the work should be widened to include the investigation of Love waves. It may turn out that certain types of flaws will have the property of converting Love waves to Lamb waves so

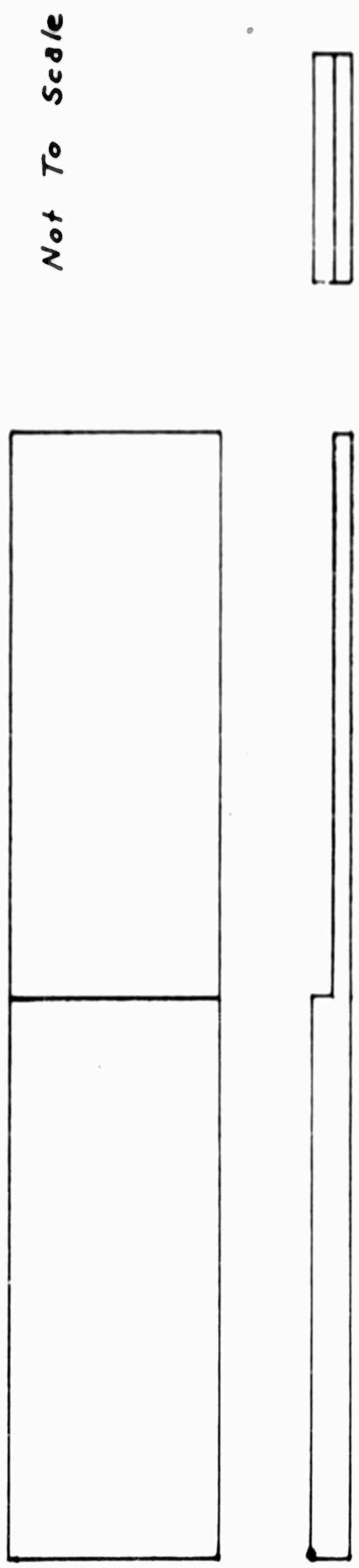


FIGURE 163

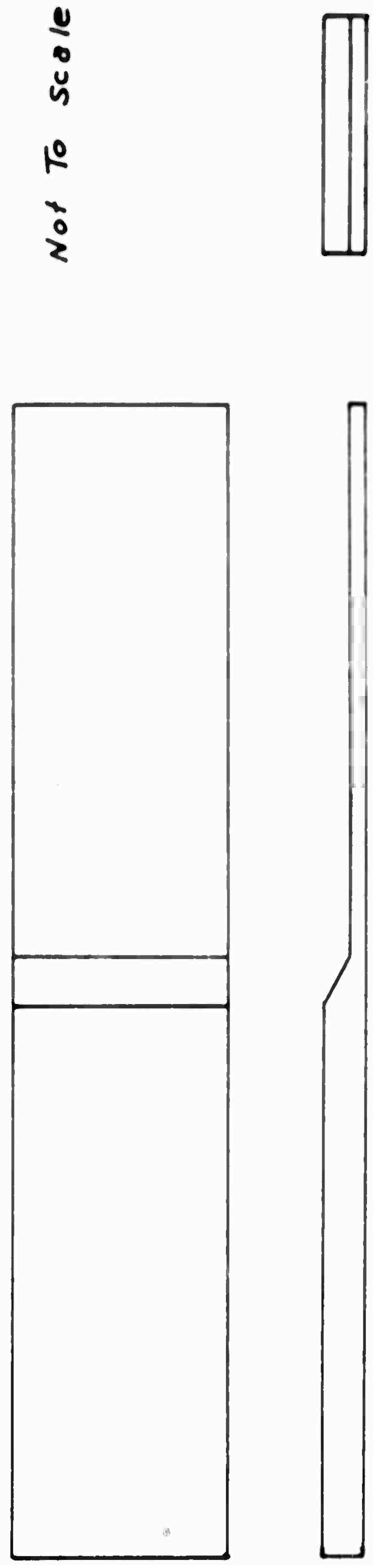


FIGURE 164

that Love waves could be generated in a plate and the detection of Lamb waves would indicate the presence of a flaw.

4.2.7 Acquisition of Various Representative Types of Covering Membranes for Missiles, Aircraft, and Space Vehicles

It is felt that as the work progresses, it will be desirable to interrupt the investigation of the basic principles of wave propagation by attempting to do testing work on representative covering membranes. Consequently, it is anticipated that contact will be made with one or more manufacturers of these materials to obtain representative types. Although the first attempts to make practical tests on these materials may fail they will in turn serve in the selection of models for the investigation of basic principles.

4.2.8 Single Transducer Method

All of the experiments conducted during this investigation were of the transmission type using two transducers. In practical testing situations, it would be desirable to use a single transducer pulse-echo system. However, the reflection of Lamb waves is accompanied, in general, by mode conversion. Consequently, when the transducer is operating as a receiver it must be sensitive to the range of phase velocities that may be encountered. The problem may be somewhat simplified if the first symmetrical mode is used at a frequency too low to permit the existence of any other modes except the first asymmetrical mode.

BIBLIOGRAPHY AND ABSTRACTS

Part one of the detailed scope of contract No. DA-23-072-505-ORD-1 is as follows: "A thorough literature survey shall be conducted to reveal the state of the ultrasonic art in so far as Lamb Waves are concerned."

Since seismologists are concerned with Lamb Waves and related phenomena both at ultrasonic frequencies (for the construction of models) and at much lower frequencies, many of these papers represent work by seismologists. It is apparent that a large potential source of ideas for the nondestructive testing of thin plates and laminated structures is present in the literature of seismology.

In the ultrasonic delay line field, there is a great deal of interest in the construction of delay lines from thin plates of metal since this permits the construction of a large amount of delay within a small volume. Since the problem of transmitting ultrasound through thin plates for nondestructive testing purposes is similar to the problem of transmitting it for the purpose of delaying a signal, the results of work on the development of thin plate delay lines have been included in this literature search.

The problem of transmitting sound through panels is very closely related to the excitation of Lamb Waves in plates, because sound is readily transmitted through panels only when it is incident at such an angle that it excites the natural modes of vibration. For this reason, the report by George B. Thurston and Raya Stern, A Bibliography on Propagation of Sound Through Plates, University of Michigan, 1959, has been very helpful and a number of abstracts were taken from this source.

Whenever the abstract prepared by the author himself was available it was used verbatim. Detailed credit was not given for authorship of the abstracts themselves as it was felt that this would detract from the usefulness of the bibliography.

American Standards Association, American Standard Acoustical Terminology

The American Standards Association definitions, standards and specifications for use in acoustical work. Sections General, Sound Transmission and Propagation, Transmission Systems and Components, Ultrasonics, Hearing and Speech, Music, Architectural Acoustics, Recording and Reproducing Underwater Sound, General Acoustic Apparatus, Shock and Vibration Acoustical Units; Index.

Arenberg, D. L. "Dispersion of High-Frequency Elastic Waves in Thin Plates," Proc. I. R. E., 46, 19 5 (Dec., 1958)

Some comments on the theory and construction of thin plate delay lines. The solution for shear waves discussed by Mapleton is extended to give the increment in velocity as a function of the order of mode, frequency, and plate thickness. Data are given on a 1000 microsecond line, 0.636 cm. thick. The effect of the different modes in causing variations in delay and insertion loss throughout the passband is discussed.

Barnes, R. B.; see Burton, C. J., and Barnes, R. B., (1949)

Bolt, R. H., see Heuter, T. F., and Bolt, R. H., (1955)

Boyle, R. W., and Lehmann, J. F., "Passage of Acoustic Waves through Materials," Trans. Roy. Soc. Can., 21, Sect. 3, 115-125.

Deals with the proportion of incident energy reflected by an infinite plate or partition of homogeneous material and finite thickness in a medium extending indefinitely in all directions when a train of sound waves is incident on one side of the plate. In some of the experiments an ultrasonic generator driven at 135,000 cycles per second is used and the medium is water and the plate lead.

Bradfield, G., "Improvements in Ultrasonic Flaw Detection," J. Brit. Inst. Radio Eng., 14, n7, 303-308, Jl '54

Three improvements are described; mode changer (to change longitudinal waves to shear waves) which is made of heavy material in form of laminated assembly; damping, loading, and circuitry of piezeolectric crystals to give improved

discrimination etc.; and devices enabling beams of mechanical waves to be steered from chosen sites to search for flaws.

Branson N. G., "Metal Wall Thickness Measured from One Side by Ultrasonic Method," Elect. Eng., v70, pp 619-623 (1951)

This article discusses the principles of metal wall thickness measurement from one side and an instrument for making such measurements. The selection of quartz crystal and range, accuracy, and limitations of the method are covered.

de Bremaecker, J. C., "Transmission and Reflection of Rayleigh Waves at Corners," Geophysics, v23, n2, Apr., 1958, pp 253-66

Methods of two dimensional model seismology were used to investigate phenomena occurring when a Rayleigh wave is incident upon a corner whose angle is comprised between 0° and 180°; wave bends its path only for angles between 130° and 180°; for smaller angles large and abrupt variations in reflection and transmission occur.

Bullen, K. E., An Introduction to the Theory of Seismology, Cambridge University Press, 1953

Very good theoretical coverage of elastic wave propagation.

Ch. V Surface Elastic Waves (Rayleigh, Love, other)

Ch. VI Reflection and Refraction of Elastic Waves

Burton, C. J.; see Schneider, W. E., and Burton, C. J., (1949)

Burton, C. J., and Barnes, R. B., "A Visual Method for Demonstrating the Path of Ultrasonic Waves Thru Thin Plates of Material," J. Appl. Phys., v 20, pp 462-467, (1949)

A visual method for studying the path of ultrasonic waves through thin plates of material has been devised, and a set of photographs showing the complexity of ultrasonic reflection and transmission by metal plates has been obtained. An attempt has also been made to estimate the critical angles of total reflection of dilation and shear waves by visual examination of the reflection of a supersonic beam. Using these data,

Young's modulus and the shear modulus for 1/16-inch aluminum sheet have been determined, and the values are compared with those obtained using a previously described electrical method and with published data.

Butler, J. B., and Vernon, J. B., "Standing Wave Technique of Thickness Measurements," J. Acoust. Soc. Am., v18, pp 212-215, (July, 1946)

Some experiments are described in which a reflectoscope was used to measure the thickness of plates varying from .0109" to 0.151" by standing wave techniques.

Carabelli, E., and Folicaldi, R., "Seismic Model Experiment on Thin Layers," Geophysical Prospecting, v5, n3, pp 317-327, (September, 1957)

Seismic two dimensional model experiments concerning reflections from horizontal layers thin in relation to wave lengths propagated through media are recorded; effects of very thin reflecting layers, individually and in combination, on reflected arrivals and influence of layers upon reflected energy in terms of their thickness.

Carlson, J. F., see Heins, A. E., and Carlson, J. F., (1947)

Carter, S. W.; see Hastings, C. H., and Carter, S. W., (1949)

Clayton, H. R., and Young, R. S., "Improvements in Design of Ultrasonic Lamination Detection Equipment," J. Sci. Instruments, v28, n5, pp 129-132, (May, 1951)

Apparatus for detection of lamination on aluminum blanks and sheets; use of transmission technique which minimizes effect of standing waves formed in sheets of specific thicknesses; how this is achieved by energizing transmitter crystal by wide band of frequencies and by providing receiver detecting all frequencies transmitted; defect of less than 3/4 in. can be detected.

Cook, E. G., Van Valkenburg, H. E., "Surface Waves at Ultrasonic Frequencies;" ASTM, Bul. n198, pp 81-84, (May, 1954)

The theory of mechanical wave propagation along the surface

of an extended solid medium and adapting it to nondestructive materials testing. Bibliography.

Cook, E. G., and Van Valkenburg, H. E., "Thickness Measurement by Ultrasonic Resonance," J. Acoust. Soc. Amer., v27, pp564-69, (May, 1955)

A mathematical analysis is presented which is applicable generally for any combination of test specimen, coupling material, and driving crystal.

Cremer, L., "Theory of Sound Damping by Thin Walls," Akust. Z., v7, pp 81-104, (1942)

The damping for perpendicular incidence is calculated and compared with the result of measurements obtained with statistical distribution of angles of incidence. Mass and frequency are the main factors. The movement of the wall is considered as a forced bending oscillation. When the component of sound velocity along the surface of the wall the speed of propagation of a free oscillation of the membrane, the pressure difference equals 0. This "coincidence effect" and its dependence on frequency is investigated, and the analogies between coincidence and resonance are discussed.

Deck, W., "Crack Detection in Aircraft Structures," Roy. Aeronautical Soc. J., v60, n551, pp 739-748, (November, 1956)

Penetrant, magnetic, electrical, X-ray, and ultrasonic methods of testing for surface and internal cracks in production and maintenance of aircraft as carried out in Swiss aircraft factories.

Dorman, J., "Numerical Solutions for Love Wave Dispersion on a Half-Space with Double Surface Layer," Geophysics, v24, pp 12-29, (February, 1959)

The IBM 650 computer of the Watson Scientific Computing Laboratory, Columbia University, was programmed to obtain numerical solutions for the period equations for Love waves on a half-space with a double surface layer. Solutions including higher modes for seven models of the continental crust-mantle system are presented. This group of related cases shows that certain properties of the solutions are diagnostic of crustal structure. These relationships are illustrated graphically.

Eisler, J. D.; see Evans, J. F., Hadley, C. F., Eisler, J. D., and Silverman, D., (1954)

Evans, J. F., Hadley, C. F., Eisler, J. D., and Silverman, D., "A Three-Dimensional Seismic Wave Model with Both Electrical and Visual Obervation of Waves," Geophysics, v19, pp 220-236 (1954)

> The short wavelengths required in a seismic model to give wavefront patterns geometrically similar to those in a large prototype (the earth) can only be obtained by using high-frequency sound waves. As sources and detectors of such high-requency waves, piezoelectric crystal are used, primarily because under identical stimuli they are capable of almost perfect duplication. Such duplication is made use of in displaying on an oscilloscope stationary patterns which are characteristic of transient particle motion at a point in the model. Also, it has made possible the direct visual observation of transient wave fronts in transparent models, techniques for which are described, and sample photographs given. As an example of quantitative use of the described model techniques, the results are presented showing symmetric and anti-symmetric wave propagation in a free elastic plate. Good agreement is found between many features of the experimental record and theoretical predictions.

Ewing, M; see Oliver, J., Press, F., and Ewing, M., (1954)

Ewing, W. M., Jardetzky, W. S., and Press, F., Elastic Waves in Layered Media, McGraw-Hill, New York, (1957)

> Very good general coverage of wave propagation theory - general principles - fundamentals. Includes many references. A comprehensive discussion of Lamb waves is found on pp 281-288.

Fay, R. D., "Interactions Between a Plate and a Sound Field," J. Accoust. Soc. Am., v20, pp 620-625 (1948)

> This paper deals with phenomena associated with the interactions between flexural vibrations in a plate and the sound field in an ambient fluid medium. An equation is developed in tractable form which determines the propagation function

of the free and forced flexural waves which can exist under stated assumptions. It is found that the reactions of the fluid on the plate makes possible a wave which cannot exist in the free plate in steady state. For a steel plate in water the potential effect of the additional wave is relatively great when the product of the frequency and the plate thickness lies between 6 and 20 kilocycle-inches.

Errata: J. Acoust. Soc. Am., v21, n3, p 272, (May, 1949)

Fay, R. D., and Fortier, O., "Transmission of Sound thru Steel Plates Immersed in Water," J. Acoust. Soc. Am., v23, pp 339-346, (1951)

The measured transmittivity of a steel plate in water is presented as a function of the angle of incidence and the product of frequency and plate thickness over wide ranges. The normal velocity of the plate surface can attain an amplitude necessary for good transmission only by constructive interference among internal reflections. It is shown that the ideal conditions can be met in a plate of finite width in only a few cases. In general, the conditions for a transmission maximum are the conditions for the existence of appropriate types of stable travelling waves in a plate of infinite extent; these conditions, however, are modified by edge effects. An apparent effect of this modification is to produce changes in the divergence of the transmitted beam and hence in the observed transmittivity

Fay, R. D., "Notes on the Transmission of Sound Through Plates," J. Acoust. Soc. Am., v25, pp 220-223, (1953)

A relatively tractable expression for the transmittivity of steel plates immersed in a fluid medium is formulated. Criteria for total transmission and total reflection are inherent in the expression.

An investigation of the discrepancies between calculated and observed transmittivity indicates a probability that the assumption of negligible losses associated with shear waves in steel is not tenable.

Finney, W. J., "Reflection of Sound from Submerged Plates," J. Acoust. Soc. Am., v 20, pp 626-637, (1948)

It has been found that an appreciable reflection occurs in the

direction of the incidence. The magnitude of this "non-specular" reflection, together with the assorted transverse vibrations in the plate, has been studied experimentally as a function of frequency, plate dimensions, and angle of incidence. Significant non-specular reflection occurs from bounded plates between about one-quarter inch and two inches thick, when the product of the thickness in inches and the frequency in kilocycles per second lies between about 20 and 60 in. -kc. Within this range, the angle of incidence for non-specular reflection, η, is described by the empirical relation $\sin \eta = 2.16/(bf)^{\frac{1}{3}}$. This relation has been found to apply also to 0.002 inch plates at 15 megacycles. The results of the present study are in agreement with a theoretical analysis by Fay, and with the experimental results obtained by Sanders at frequencies of several megacycles.

Firestone, F. A., and Ling, D. S., Report on the Propagation of Waves In Plates - Lamb and Rayleigh Waves, Sperry Products, Inc., Tech, Rep. 50-6001, Danbury, Connecticut (1945)

The various inspection problems to which Lamb waves might be applied are discussed. The different modes are discussed with respect to group velocity, phase velocity, particle motion and means of generation. Calculations were made and curves plotted of group velocity and phase velocity as functions of frequency-thickness product and wavelength-to-thickness ratio for aluminum. Particle motion paths are shown for various modes and wavelengths. The spectrum of the pulsed sine wave and its relation to Lamb wave dispersion is discussed. A method of measuring plate thickness by utilizing interference between the first symmetrical and the first anti-symmetrical modes is described. There is evidence of mode conversion upon reflection at an edge of the plate. The design of transducers for exciting Lamb waves in plates is duscussed.

Firestone, F. A., "The Supersonic Reflectoscope, an Instrument for Inspecting the Interior of Solid Parts by Means of Sound Waves," J. Acoust. Soc. Am., v 17, (1946)

The reflectroscope is described and its principle of operation is explained. Figures show typical reflectoscope presentations when reflectoscope is used for flaw detection, wall thickness measurement, lamination detection, bond determination, grain size measurement, and weld testing.

Firestone F. A., and Frederick, J. R., Refinements in Supersonic Reflectoscopy: Polarized Sound," J. Acoust. Soc. Am., v18, pp 200-211, (1946)

One form of simple circuit for the generation of the short duration high frequency voltage trains used in the supersonic reflectoscope is shown. The required band width of the system including the crystal, is discussed. Techniques for the radiation of longitudinal shear and Rayleigh waves are set forth. Either longitudinal or shear waves can be used to establish various modes of thickness resonance through a plate for measuring velocity of propagation or thickness. Shear waves have some of the properties of polarized light for instance, double refraction has been measured by three methods in elastically aeolotropic solids and a technique for the direct indication of the amount of elastic aeolotropy developed. By means of a quarter wave plate circularly polarized sound can be produced

Firestone F., Tricks with the Supersonic Reflectoscope, Journal of Nondestructive Testing, v7, n2 pp 5-19 (Fall 1948)

While a brief explanation is made of the principle of the supersonic reflectoscope as applied to its everyday function of detecting flaws in the interior of metal parts by observing the reflections of a short train of high frequency sound waves from the flaws, the emphasis is here placed on the unusual and unexpected possibilities of the reflectoscope as indicated in the section headings of this paper: Controlling the Divergence of the Sound Beam Detecting Flaws Lying Near the Surface with the Schnozzle, Increasing the Strength of High Frequencies with Tinfoil Under the Crystal, Measuring the Average Grain Size in Metals, Sending Waves Approximately Parallel to Surface of Plate, Generating a Strip of Lamb Waves for the Testing of Thin Sheets, Determining Residual Stress or Incipient Fatigue Beneath a Surface by Measuring the Velocity of Surface Waves, Flaw Detection or Grain Size Determination Near the Surface, Generating Shear Waves or Longitudinal Waves by Reflection; Determining Poisson's Ratio, Young's Modulus, and the Shearing Modulus, by Measuring the Velocity of Longitudinal and Shear Waves; Accurately Measuring the Thickness of a Metal Sheet whose Opposite Side is Inaccessible by Resonance; Measuring Elastic Aeolotropy with Shear Waves.

Firestone, F. A., and Ling, D. S., "Methods and Means of Generating and Utilizing Vibrational Waves in Plates." United States Patent No. 2,536,128, January 2 1951.

Lambs calculations are summarized. Curves are presented giving the phase velocities of Lamb waves in aluminum plates as function of frequency-thickness product. Methods and devices are described for inducing Lamb waves in plates. Methods are described for testing plates for defects by mode conversion of Lamb waves. A method is described for determining the thickness of a plate by utilizing the interference between two different modes of slightly different wavelength.

Folicaldi, R.; see Carabelli, E., and Folicaldi, R., (1957)

Fortier, O., see Fay, R. D., and Fortier O., (1951)

Frederick, J. R., see Firestone, F A., and Frederick J. R., (1946)

Gericke O. R., Ultrasonic Methods for Near Surface Flaw Detection Watertown Arsenal, Watertown, Mass., 1959

For the nondestructive inspection of materials with ultrasonic waves, today the "pulse echo" method is widely utilized. Compared to through transmission testing this method has the advantage that access to only one surface of the test piece is required.

Pulse echo testing has certain limitations with regard to defects that are located close to the test surface, however, due to the finite length of the ultrasonic pulse. If the transit time of the initially transmitted pulse, for traveling from the ultrasonic transducer to the discontinuity and back, becomes shorter than the pulse length, the echo from the flaw will interfere with the transmitted pulse. Consequently an indication of the flaw cannot be obtained.

In order to detect near-surface defects, either extremely short ultrasonic pulses or an artificial pulse delay are required. The objective of the work covered by this report was to investigate pulse delay and develop methods which can be used for practical applications.

Three of the procedures which are discussed show promising experimental results. Defects as close as 0.015" to the test surface can be detected, although fairly long pulses of 3 to 10 microseconds duration are used.

Goldman, S., Frequency Analysis, Modulation and Noise, McGraw-Hill New York, (1948)

The first half of this textbook is a discussion of the frequency analysis of periodic and nonperiodic waveforms by means of the Fourier Series and the Fourier Integral respectively. The treatment presupposes a knowledge of calculus. Among the topics treated are the principle of stationary phase, phase velocity and group velocity, and reciprocal spreading.

Goodman, L. E., "Circular-Crested Vibrations of An Elastic Solid Bounded by Two Parallel Planes," Proceedings of the First U. S National Congress of Applied Mechanics, American Society of Mechanical Engineers, New York, N.Y. pp 65-73, (1951)

Two fundamental radially symmetric solutions of the linear equation of elasticity are found to describe free vibrations of an elastic plate of finite thickness. It is shown that they correspond to 'extensional' and 'flexural' motions and are analogous to the straight crested (plane strain) waves discovered by Lamb and Rayleigh. At large distances from the origin the circular-crested and straight crested motions become indistinguishable.

Following the treatment of Lamb, the transcendental equation relating the frequencies of vibration to the wave shape is developed. A numerical tabulation of the solutions of the frequency equation is given and the shapes of the modes are described in detail. For a particular set of modes there exist surfaces across which no stress is transmitted. These surfaces define shapes for which the motion in question represents a mode of free vibration.

In the limit, for very thick plates, the solutions are shown to correspond to Rayleigh surface waves, while for thin plates, they approach asymptomatically, well-known results in elementary plate theory.

Gotz, J., "The Passage of Sound Through Metal Plates Immersed in Fluids, When the Waves are Plane and are Incident Obliquely," Akust. Z., v8, pp 145-168, (1943) (In German)

I. The origin of sound transmission problems and their use in the testing of materials.

1. Rules of use and the attempt of the solution of the problems.

2. The meaning of sound transmission problems by the detection of flaws in plates. Description of measuring apparatus.

3. Aim of the work.

II. The transmission of plane sound waves through plane parallel partition.

1. The problem

2. The theory of Rayleigh for media free from shearing stress.

3. The theory of Reissner and the work of the Zurich group.

4. The sound damping by simple free partitions in room and building acoustics. The coincidence of bending waves.

5. The attempt to use the theory of the coincidence of bending waves in the transmission of sound through metal plates in a liquid.

6. Attempt the exhibition of vibrating plates as bending waves under consideration of the rotational inertia and the shearing deformation.

7. Determination of the bending wave decrement through treatment of the problems as two dimensional wave problems.

8. The sound permeability by the incident angle within the angle of total reflection. The thickness resonance.

9. Construction of thickness resonance by layers of partitions free from shearing stress as coincidence.

10. Agreement of Reissner's theory with the coincidence theory by solid free partitions.

III. Attempt with the arrangement described under I.

1. The maximum sound decrement.
2. The progress of the tangential wave outside the defined sound ray.
3. Summary.

Hadley, C. F.; see Evans, J. F., Hadley, C. F., Eisler, J. D., and Silverman, D., (1954)

Hart, S. D., see Osborne, M. F. M., and Hart, S. D., (1945)

Hart, S. D.; see Osborne, M. F. M., and Hart, S. D., (1946)

Hastings, C. H., and Carter, S. W., "Inspection, Processing, and Manufacturing Control of Metals by Ultrasonic Methods, ASTM Special Publication No. 101, pp 14-61, (1949)

A survey of the entire field of applied ultrasonics with emphasis on nondestructive testing.

History, Ultrasonic Generators Galton Whistle, Hartman Gas Current Generator, Holtzmann Mechanical Generator Thermal Generators, magnetostriction generator, piezoelectric generators. Application non-metallurgical, metallurgical, powder metallurgy, grain size refinement, acceleration of solidification, alloying, heat treating, fatigue testing, plating, bonding, tinning, degassing of melts. Inspection detection of discontinuities, theoretical considerations, acoustic wave velocity, energy distribution, boundary phenomena, refraction, reflection, impedance matching, internal discontinuities, principles of testing, injection of ultrasonics, detection of ultrasonics, reflection method, interpretation of results, instrumentation, transmitter requirements, receiver requirements, requirements for visual presentation, general features of circuits, thick sections, strips, slabs, plates, bars, lack of bond, welds and weldments, thickness meaasurement.

Healy, John; see Press, Frank, and Healy, John (1957)

Heins, A. E., and Carlson, J. F., "The Reflection of an Electromagnetic Plane Wave by an Infinite Set of Plates," Quart. Appl. Math., v4, pp 313-29 (Jan., 1947)

A plane wave is incident upon an infinite set of staggered, equally spaced, semi-infinite metallic plates of zero thickness. Plane waves are reflected in certain directions (depending upon the geometry of the system) and these are investigated by an integral equation method. The equation is solved by means of a Fourier Transform. Expressions for the reflection coefficients are obtained and the far field is investigated.

Heins, A. E., and Carlson, J. F., "The Reflection of an Electromagnetic Plane Wave by an Infinite Set of Plates," Quart. Appl. Math., v5, pp 82-8, (April, 1947)

A continuation of previous work. A Fourier transform method is used. The magnetic field of the incident wave is taken parallel to the edges of the plates. The reflection and transmission coefficients are calculated. These are independent of the wavelength of the incident wave.

Heuter, T. F., and Bolt, R. H., Sonics, Wiley, New York, (1955)

A survey of the field of sound and ultrasound. Emphasis is on industrial and laboratory techniques. Chapter headings are Introduction, Basic Principles, Radiation, Piezoelectric Transducers, Magnetostrictive Transducers, Physical Mechanisms for Sonic Processing, Devices, and Techniques for Sonic Processing. Principles of Sonic Testing and Analysis, Acoustical Relaxation, Mechanisms in Fluids. Transmission thru plates pp 75-6; Velocity measurements with thin plates using coincidence principle pp 345-346.

Holden, Q. N., "Longitudinal Modes of Elastic Waves in Isotropic Cylinders and Slabs," Bell System Technical Journal, v30, pp 956-969, (1951)

The general properties of the longitudinal modes in cylinders and slabs are developed with the aid of the close formal analogy between the dispersion equations for the two cases.

Jardetzky, W. S.; see Ewing, W. M., Jardetzky, W. S., and Press, F., (1957)

Kane, T. R., "Reflection of Flexural Waves at the Edge of a Plate," Columbia University - Dept. Civ. Engr. - Techn. Report n6, Feb., 1953, 22 p, 8 supp plates. See also ASME Paper n53-A-42 for meeting Nov. 29 - Dec. 3, 1953; 8 p.

Reflection of straight-crested flexural waves at edge of semi-infinite plate studied in terms of 2-dimensional plate theory; flexural wave gives rise to three reflected waves, two flexural waves and shear wave, number of special cases, involving degenerate forms of these motions, are investigated.

Kinsler, Lawrence E., Fundamentals of Acoustics, Wiley, New York, 1950

A textbook for graduate students covering the fundamental principles underlying the generation, transmission, and reception of acoustic waves. Analysis of the various types of vibration of solid bodies and of the propagation of sound waves through fluid media. A limited number of applications of acoustics.

Kleint, R. E., see Varnye, Z. J., and Kleint, R. E., (1957)

Knopoff, L., "Small Three-Dimensional Seismic Models," Am. Geophysical Union Trans., v36, n6, pp 1029-34, (Dec., 1955)

Fundamental theory of models is reviewed and application is made to the case of 3-dimensional laboratory models for studying seismic phenomena; the nature of useable transducers and media are considered from the standpoint of bandwidth restrictions imposed by the modeling method; experimental procedure is described and results for a model of Lamb's problem obtained; influence of imperfections; results for wax material.

Knopoff, L., "Surface Motions of a Thick Plate," J. Appl. Phys., v29, pp 661-670, (1958)

The motion of a thick elastic plate, infinite in two dimensions and bounded by air on the two parallel faces, has been studied. The plate is excited at a point on one face, as in Lamb's problem, by a force impulsive in time, and directed normal to the surface.

Kolsky, H., Stress Waves in Solids, Clarendon Press, Oxford, (1953)

Part I Treats the propagation of stress in perfectly elastic solids and the theory is developed as a mathematical consequence of Hooks's law and the equations of motion. The only difference between individual solids in this treatment results from differences in the values of their elastic constants and densities. Recent experimental work concerned with the verification of the theory is described.

Part II Concerned with the propagation of stress waves through solids which are not perfectly elastic. The measurement of internal friction and the nature of the various dissipative processes which cause it are discussed first. A review of experimental work on the measurement of dynamic elastic properties is then given. Finally, the theory of plastic waves and shock waves is outlined and some of the fracture phenomena produced by large stress pulses are described. A brief description of Lamb's treatment of waves in an elastic plate is given on pp 81-83.

Lamb Wave Search Unit" Sperry Products, Inc., Danbury, Connecticut, October 1958; Sales Data Sheet 50-227, (Oct. 1958)

Describes the Sperry search unit; photograph.

Lamb, G. L., Diffraction of a Plane Sound Wave by a Semi-Infinite Thin Elastic Plate, Acoustics Laboratory, Massachusetts Institute of Technology, Cambridge, Mass., 1959, AD-221-193 div. 25; Reprint from Jnl. of the Acoustical Society of America, v 31, 929-935, (July 1959)

The diffraction of a plane small amplitude sound wave incident upon a semi-infinite thin elastic plate is investigated theoretically. The problem is formulated in terms of an integral equation relating the discontinuity in pressure across the diffracting plate to its flexural displacement and the usual fourth order thin plate differential equation governing the flexural motion of the plate when driven by the pressure discontinuity. This pair of coupled equations is then shown to be amenable to solution by the Wiener-Hopf method. A perturbation procedure, valid in the limit of increasing plate stiffness, is employed to obtain expressions for the sound fields radiated by and transmitted through the plate as well as for the diffracted sound field.

Lamb, H., "On Waves in an Elastic Plate," Proc. Roy. Soc. (London), v93, pp 114-128, (1917)

An examination of the problem of two-dimensional waves in a solid bounded by parallel planes.

The theory of surface (Rayleigh) waves has been extended by H. Lamb to the case of a solid body bounded by two parallel planes.

Lambert, L., Research on Ultrasonic Delay Lines, Electronics Research Labs., Columbia University, New York (1959), AD-219 634 Div. 8, 25 (31 July, 59)

The general theory of elastic wave propagation is briefly presented. Reflection and refraction of plane elastic waves at plane interfaces are studied with special emphasis on the solid-air interface. Solutions to the wave equation for solid plates are presented and the specific solutions of interest in the design of delay lines are studied in detail. Design information is obtained for the vertical dimension of transducers to minimize the excitation of spurious modes in plates, thus reducing ripples in the passband of a delay line. Expressions permitting the calculation of delay variation with frequency are included and plotted for typical delay line parameters. Various kinds of quartz transducers are discussed. An equivalent circuit for a thickness vibrating piezoelectric transducer is presented and the implications of the circuit investigated. Expressions are obtained for midband loss and frequency response. The possibility of double-peaking in the frequency response is discussed, and the condition necessary for its occurrence determined. This includes results for typical delay line parameters. Various techniques of bonding are discussed and the results of one technique are compared to the performance predicted by theory.

Leep, R. W.; see Ross, J. D., and Leep, R. W., (1957)

Lehfeldt, W., "Ultraschall-Pruefung von Blechen and Baendern," Elektronische Rundschay, v9, n4, pp 135-8, (Apr., 1955)

Ultrasonic testing of sheet and strip metals preferably done by traverse radiation, its application requires use of devices for

continuous passage of test objects between sound transducer heads; installation in producer's rolling plant requires smaller investment than mounting of special units at consumer's plant; method is also applicable to deflection oscillations and vibrations.

Lehmann, J. F.; see Boyle, R. W., and Lehmann, J. F.

Ling, D. S.; see Firestone, F. A., and Ling, D. S., (1954)

Ling, D. S.; see Firestone, F. A., and Ling, D. S., (1951)

Love, A. E. H., Some Problems of Geodynamics, University Press, Cambridge, (1911)

Chapter XI Theory of the Propagation of Seismic Waves. Dilational and distortional waves. First interpretation of seismic records. Rayleigh-waves. Lamb's theory of tremors. Transverse movement observed in main shock. Methods of investigation. Theory of transmission of waves through gravitating compressible body. Dilatational wave accompanied by slight rotation. Dependence of wave-velocity upon locality and wave-length. Effects of dispersion. Theory of superficial waves on a sphere. Advancing wave as aggregate of standing oscillations. Correction to velocity of Rayleigh-waves on account of gravity. Theory of transverse waves in superficial layer. Condition that the waves may be practically confined to the layer. Dispersion. Subjacent material of smaller rigidity ineffective. Theory of the equation for wave-velocity. Sense of alteration through discontinuity at bottom of layer. Numerical example. Ratio of horizontal and vertical displacements. Discussion of conditions for the existence of a second type of superficial wave. General discussion of the main shock.

Lyamshev, L. M., "Reflection of Sound by a Thin Plate in Water," Dokl. Akad. Nauk SSSR, v99, pp 719-721 (1954) (translated from Russian by M. D. Friedman, 572 California Street, Newtonville, Mass., 4pp)

Observations using high-frequency pulse technique are described briefly. Reflections in a direction anti-parallel to the obliquely incident ray are found at two critical angles, corresponding, respectively, to flexural and to compressional waves in the plate. The latter case is presented as new; related transmission data are not cited or discussed.

Lyamshev, L. M., "Diffraction of Sound Upon a Thin Bounded Plate in Liquid," Sov. Phys. - Acous., v1, pp 145-151, (1955)

The paper examines the problem involved in the diffraction of sound upon a thin bounded plate in liquid; the analysis takes into account the longitudinal and flexural vibrations of the plate.

It is shown that at certain angles of incidence the passage of the sound through the plate is observed, as well as intense scattering in a direction opposite to the direction of the incident wave (this constitutes so-called non-mirror-like reflection). This paper is entirely theoretical.

Lyamshev, L. M., "Nonspecular Reflection of Sound from Thin Bounded Plates Submerged in Liquid," Acta Phys. Hungar., v6, pp 33-65, (1956) (In Russian)

Discovery is made of a nonspecular ultrasonic reflection from thin finite plates under water, occurring at considerably smaller angles of incidence than that due to coincidence effect in bending. Precise measurements made in the Fraunhofer zone of reflection, using metal plates up to 1.1 mm thick and up to 6 × 6 cm., with megacycle sound waves, show this phenomenon to occur when the phase velocity of the incident waves along the plate is equal to the speed of longitudinal waves in the plate. A detailed theoretical analysis is made of sound diffraction, taking into account this plate response. The theory appears to describe the observed results satisfactorily.

Lyon, R. H., "Response of an Elastic Plate to Localized Driving Forces" J. Acoust. Soc. Am., v27, pp 259-265, (1955)

The vibration of an infinite elastic plate when driven by a localized driving force is studied theoretically. The dilation and shear potentials are expressed as Fourier integrals, the boundary conditions are applied, and the integrals evaluated by the calculus of residues. It is found that the motion of the plate may be represented by a discrete sum of nonorthogonal eigenmodes. These modes represent two types of waves: propagated and attenuated. The former are obtained from previous work on coincidence transmission. The latter are calculated by a graphical method, and presented in such a manner that they tie in continuously with the propagated modes. Certain unresolved features of the problem are discussed. One previous application of the theoretical results is disclosed.

Makinson, K. R., "Transmission of Ultrasonic Waves Through a Thin Solid Plate at the Critical Angle for the Dilational Wave," J. Acoust. Soc. Am., v 24, pp 202-206, (1952)

The transmission, through an isotropic solid plate immersed in a liquid, of ultrasonic waves incident at the critical angle for total reflection of the dilatational wave, is examined experimentally and theoretically. It has previously been held that total reflection does not occur at this angle when the thickness of the solid is less than or comparable with the wavelength of the dilatational wave in it. It is now shown that owing to interference between the dilatational and rotational waves, total reflection does occur at or very near this angle for a considerable range of thickness in this region. It is therefore possible to use the total reflection method for the determination of the dilatational velocity in a solid even when only thin specimens of the solid are available.

Makinson, K. R., "Transmission of Sound Thru Plates," J. Acoust. Soc. Am., v25, pp 1202, (1953)

Fay's treatment of the theoretical problem of the transmission of sound through a solid plate immersed in a fluid is used to deduce an additional criterion for total reflection, and some comments are made on the criteria for total transmission.

Mapleton, R. E., "Elastic Wave Propagation in Solid Media," J. Appl. Phys., v23, pp 1346-1354, (December, 1952)

The reflection of elastic waves at plane solid-vacuum interfaces and the two-dimensional solutions of the vector equation of equilibrium for elastic waves propagating through elastic plates of uniform thickness are investigated. Both the isotropic solid and the cubic system of non-piezoelectric single crystals are treated. The objective is to compare multiple-reflection solid delay lines, isotropic versus single crystal, in terms of thickness of the plate required for satisfactory propagation and reflection characteristics. The sections on wave propagation are not exhaustive, but consider the particular wave mode that most closely resembles the plane shear wave with displacement component normal to the face of the plate. In the sections on single crystals, it is shown that plane shear waves propagate with a speed independent of the direction provided that the displacement is parallel to a crystallographic axis. The reflection

laws for these plane shear waves are the same for isotropic solids and single crystals provided that the displacement is contained in a plane that is parallel to the stress-free interface. A calculation compares the wave propagating characteristics of plates of fused quartz and calcium fluoride for operation at 15 mcs. The plate of calcium fluoride should be roughly three times thicker to give the same performance as the plate of fused quartz.

Mason, W. P., Physical Acoustics and the Properties of Solids, D. Van Nostrand Company, Inc., Princeton, New Jersey. (1958)

Part I: Basic principles with engineering applications: discussion of instruments for measuring attenuation, velocity, characteristic impedance, application to mechanical filters, drilling, soldering, flaw detection and delay lines, and to the study of fatigue, fracture, wear, cutting and grinding.

Part II: Analytical uses: phenomenological models, thermal damping, domain motion effects, grain scattering, interstitial diffusion, dislocation effects, transmission in single crystal quartz and glasses, damping of free electrons.

McGonnagle, W. J., "Ultrasonic Shear Wave Testing," Metal Progress, v70, n4, pp 97-99, (October, 1956)

Successful application of ultrasonic equipment for flaw detection in pipes and tubes; difficult-to-reach sections can be inspected using shear waves rather than longitudinal waves normally employed; results obtained with various artificial defects using this technique.

Meres, M. W.: see Muskat, M., and Meres, M. W., (1940)

Meyer, E., "Survey on Sound Transmission in a Body by Models," Akust-Bei., v1, pp 51-58, (1956) (In German)

After a short survey over the different wave forms in solids and the experimental methods for the investigation of the propagation of structure-borne sound in models, the following subjects are treated: Measurements of sound velocity and damping of flexural waves; propagation of flexural waves around corners; effect of soft intermediate layers; input impedance of plates for excitation at one point, radiation of plates excited to flexural vibrations.

Mindlin, R. D., An Introduction to the Mathematical Theory of Vibrations of Elastic Plates, U. S. Army Signal Corps Engineering Laboratories, Ft. Monmouth, New Jersey, (1955), Signal Corps Contract DA-36-039 SC-56772.

This monograph is a mathematical treatment of the vibrations of elastic plates which in general are finite and non-isotropic. Lamb waves are included as the simpler case of an isotropic, infinite, elastic plate in vacuo on pp. 2.32-2.40.

Moriarty, C. D., "Ultrasonic Flaw Detection in Pipes by Means of Shear Waves," ASME Trans., v73, pp 225-229, Discussion pp 229-235, (April, 1951)

This paper describes a procedure by which ultrasonic methods can be applied to the inspection of high pressure piping. A description of the equipment used is included together with some basic information by which the application to general pipe inspection can be evaluated. In the interest of accenting the application phases, a theoretical discussion of ultrasonic waves and their transformations has been omitted.

Murdoch, A. M., and Van Valkenburg, H. E., Ultrasonic Equipment for Inspection of Bonded Sheet Metal Aircraft Components, Sperry Products, Inc., Danbury, Connecticut, 1959; Sperry Report TR-083, Contract AF 33 (600)-32131 Final Report

New equipment for nondestructive testing of bonded aircraft components using a refinement of the ultrasonic reflectoscope principle was developed and evaluated. Acoustic pulses of the specified duration (less than 0.25 microsecond) were successfully generated and detected by means of a recently developed technique employing unidirectional impulses rather than oscillatory wave-trains. With optimum adjustment aluminum thicknesses less than .020 inch were resolved. Over 100 samples, adhesive bonded lap joint and honeycomb, and brazed honeycomb representing a wide range of bond conditions were investigated and results correlated with other ultrasonic and destructive tests. Detectable faults in 2 ply lap joints included voids, overage adhesive, low pressure bond, undercure, and partial contact. Poor adhesion due to unclean surfaces was not detectable. Results on honeycomb specimens were not satisfactory. Design and construction of equipment applicable to production inspection were not undertaken.

Lamb wave method discussed on pp 32-33.

Muskat, M., and Meres, M. W., "Reflection and Transmission Coefficients for Plane Waves in Elastic Media," Geophysics, v5, n2, pp 115-148, (1940)

The results are given of a systematic series of calculations on the coefficients of reflection and transmission for plane waves incident on elastic interfaces. Tables are given for the amplitudes of the reflected and transmitted longitudinal and transverse waves, for the intensities of these components and for the fractions of the incident energy carried away by them. For incident longitudinal waves calculations were carried out for angles of incidence between 0 and 30° with 5° intervals. For incident transverse waves polarized in the plane of incidence results are given for four angles of incidence up to approximately 16°. For incident transverse waves polarized normal to the plane of incidence the calculations were carried through for all angles of incidence - in steps of 5° - up to total reflection. All the calculations were carried through for interfacial density ratios of 0.7 to 1.3 in steps of 0.1, and interfacial velocity ratios between 0.5 and 2.0 in steps of 0.25.

Muskat, M., and Meres, M. W., "The Seismic Wave Energy Reflected from Various Types of Stratified Horizons," Geophysics, v5 n2. pp 149-155.

Two applications are made of the reflection and transmission coefficients reported in the preceding paper. These concern the effect of the angle of incidence upon the fraction of incident energy returning to the surface, and the effect of velocity stratification upon the energy return.

Nishimura, G., see Sezawa, K., and Nishimura, G.

Niwa, N., "Cathode Ray Tube Type Ultrasonic Thickness Gauge and Its Application to Nondestructive Inspection " Univ. of Tokyo-Inst. Indus. Science Report, v7, nl, pp 1-67, (February, 1958)

Principles of nondestructive inspection by resonance methods; thickness curves of some materials traced and compared with each other, design of gauge; scaling system; design of probes; testing standard test plates; thickness and sound velocity measurement; examples of detecting defects in thin metal plates. 44 refs. (In Japanese with English abstract)

"Non-Destructive Testing," Aircraft Production, v20, n8, n9, n11, n12, August, 1958: pp 314-322, September; pp 358-368, November: pp 420-431, December; pp 464-482.

August: Basic approach to ultrasonic methods of flaw detection of raw materials and techniques, used at Production Inspection Department of Vickers-Armstrongs (Aircraft) Ltd; correlation of data and coding of defective material. September: Ultrasonic examination of wing skin panels; use of magnetic crack detection techniques. November: Use of dye and fluorescent penetrants. December: Inspection; examination of aircraft structures.

Olha, W. R.; see Van Valkenburg, H. E., Wilson, R. A., and Olha, W. R.

Oliver, J., Press, F., and Ewing, M., "Two-Dimensional Model Seismology," Geophysics, v19, n2, pp 202-219, (April, 1954)

The solutions of many problems in seismology may be obtained by means of ultrasonic pulses propagating in small scale models. Thin sheets, serving as two-dimensional models, are particularly advantageous because of their low cost, availability, ease of fabrication into various configurations, lower energy requirements, and appropriate dilational-to-shear velocity ratios. Four examples are presented: flexural waves in a sheet, Rayleigh waves in a low velocity layer overlying a semi-infinite high velocity layer, Rayleigh waves in a high velocity layer overlying a semi-infinite low velocity layer, and body and surface waves in a disk.

Oliver, J.; see Press, F., and Oliver, J., (1955)

Oliver, J., "Body Waves in Layered Seismic Models," Seismological Soc. Am. Earthquake Notes, v27, n4, pp 29-38, (December, 1956)

Application of seismic model technique to some simple layered configurations; these include; single layer with both top and bottom surface free and bottom bounded, first by lower velocity material of semi-infinite extent; and second by higher velocity material of semi-infinite extent; single layer whose velocity is continuous function of depth.

Osborne, M. F. M., and Hart, S. D., "Transmission, Reflection, and Guiding of an Exponential Pulse by a Steel Plate in Water, I. Theory," J. Acoust. Soc. Am., v17, pp 1-18, (1945)

The problem of the interaction of an underwater explosion wave with a steel plate is discussed, particularly those aspects in which the plate can be considered as an elastic, 2-dimensional wave-guide. The phase velocities of the more important modes of the plate are evaluated as functions of the frequency. They are used to derive the properties of the precursor, an oscillation which precedes the explosion wave as it travels along the plate. The results of the theory are compared with experiment. The methods are applicable to the evaluation of the phase velocities of the modes of an electromagnetic-wave guide. The propagation of a transient, such as an explosion wave, down a wave guide, presents the mathematical problem of the evaluation of a contour integral of a function of a complex variable defined implicitly. No rigorous solution as yet exists.

Osborne, M. F. M., and Hart, S. D., "Transmission, Reflection and Guiding of an Exponential Pulse by a Steel Plate in Water," J. Acoust. Soc. Am., v18, pp 170-184, (1946) (Naval Research Lab. Rept. S-2564 and S-2564-814270 PB 17570 and 16085)

In this paper are given the results of an experimental study of the interaction of an explosion wave with a water-backed steel plate. Data are given showing the dependence of the transmitted and reflected waves on the angle of incidence, and of the diffracted wave on the position behind the plate. The plate acts as a filter, removing the high frequencies from the transmitted wave and the low frequencies from the reflected wave. The reflected wave is approximately constant in shape or time scale, with varying angle of incidence. Its amplitude has a broad maximum at normal incidence. In addition to the reflected, transmitted, and diffracted waves, waves can travel along the plate, in which case the plate acts as a wave guide. As a consequence of the dispersion of the guided waves, a precursor precedes the explosion wave as it travels along the plate. The dependence of the frequency, length, and amplitude of this precursor upon orientation of the plate, position and time has been determined.

Petrashen, G. I., "O nekotorykh interferentsionnykh yavleniyaka v dvukhsloinoi srede," Akademiya Nauk SSSR, Izvestiya, Seriya, Geofizicheskaya, v20, n10, pp 1219-1231, (October, 1957)

Some interferential phenomena in two-layer medium; study of the process of formation of interference waves in a thin stratus which is in contact with a semispace, assuming that source of oscillation and seismographs are located on the surface.

Postma, G. W., "Wave Propagation in Stratified Medium" *Geophysics,* v20, n4, pp 780-806, (October, 1955)

Wave equation is derived from stress strain relations and equation of motion, graphical procedure for derivation of characteristic velocities from five elastic moduli and average density of medium, concept of wave surface; possible application of theory to actual field problems.

Press, F.; see Oliver, J., Press, F., and Ewing, M., (1954)

Press, and Healy, *Absorption of Rayleigh Waves in Low-Loss Media,* Seismological Lab., Calif. Inst. of Tech., Pasadena; (1958) 4 p. incl. illus. (Contribution No. 838), Contract DA 04-495-ORD-847, Project TB2-001; AD-137 029; Div. 2 10, 25 1, Unclassified report.

Available by reference to Jnl. of Applied Physics 28:1323-1325, Nov. 57. Many materials are characterized by internal dissipation parameter 1/Q. An expression is derived for such media relating the Rayleigh wave absorption coefficient to compressional and shear wave absorption coefficients with the elastic velocities as parameters. Ultrasonic experiments are described in which the three absorption coefficients are measured in thin Plexiglas sheets. The theoretically derived expression satisfactorily relates the observed absorption coefficients.

Press, F., and Oliver, J., "Model Study of Air-Coupled Surface Waves," *J. Acous. Soc. Am.,* v27, pp 43-46, (January, 1955)

Use of flexural waves generated in thin plate by spark source to investigate properties of air coupled surface waves; experimental study of effects of source elevation, fetch of air pulse, and cancellation by destructive interference.

Press, F., "Seismic Model Study of Phase Velocity Method of Exploration," *Geophysics,* v22, n2, pp 275-285, (April, 1957)

Variations in phase velocity of earthquake generated surface waves used to determine local variations in thickness of earth's crust; seismic model study of effect of thickness changes, lithology changes, faults and scarps, on phase velocity of surface waves.

Rayleigh, J. S., Theory of Sound, Dover, New York (1894)

A treastise covering the entire field of sound. Chapter X deals with the vibrations of plates.

Rayleigh, J. S., "On Waves Propagated Along the Plane Surface of an Elastic Solid," Proc. Lon. Math. Soc., v17, pp 4-11, (1895)

The behavior of waves upon the free surface of an infinite homogeneous isotropic elastic solid is investigated in this paper. These waves are confined to a superficial region the only disturbance being found within a distance comparable to the wavelength. (Rayleigh Waves)

Reissner, H., "Transmission of Compression Waves Through a Solid Plate Immersed in a Fluid, " Helv. Phys. Acta, v11, pp 140-155 (In German)

Assuming plane compression waves falling on a parallel-sided plate in which there is no internal friction, the author deduces an expression for the transmission, i.e., the ratio of the squares of the amplitudes of the incident and emergent waves. This expression is a function of the angle of incidence, the angles of refraction in the plate, the wave lengths, and the velocities of both compression and torsional waves in the plate. It is useful in determining the elastic constants of the plate. Its possible value in the theory of sound insulation of walls is pointed out.

Richardson, E. G., "Acoustics in Relation to Radio Engineering," Brit. Inst. Radio Eng. J., v12, n11, pp 577-84, (Nov., 1952)

Analogies between acoustic and electromagnetic phenomena; ideas of acoustic impedance resonators, filters, transmission lines, waveguides and radiators, discussed in relation to comparable electromagnetic cases; phenomena experienced in application of "sonar" and sound ranging are compared with propagation effects experienced in ionosphere by electromagnetic waves; ultrasonic waves.

Rizinichenko, U. V., and Shamina, O. G., "Ob uprugikh volnakh v tverdoi sloistoi srede po issledovaniyam na dvukhmernykh modelyakh," Akademiya Nauk SSSR, Izvestiya, Seriya Geofizicheskaya, v20, n7, pp 855-73, (July, 1957)

Elastic waves in solid stratified medium, according to studies of two dimensional models, ultrasonic method of exciting waves was applied; kinematic and dynamic characteristics of waves.

Ross, J. D., and Leep, R. W., "Ultrasonic Transmission Tester," Non-Destructive Testing, v15, pp 152-4, (May-June, 1957)

Tester developed at Savannah River Laboratory; usual high-voltage pulser and high gain amplifier were eliminated by taking advantage of greater coupling coefficient and lower driving impedance of barium titante transducers; ultrasonic pulse is transmitted through object under inspection and received pulse is checked for abnormal attenuation; abnormally attenuated pulses can be counted as object is scanned.

Sanders, F. H., "Transmission of Sound Through Thin Plates," Canad. J. of Research, v17, Sect. A, pp 179-193, (Sept., 1939)

The transmission of sound through plates of brass and nickel has been studied for angles of incidence ranging from 0 to 70° using effective plate thicknesses varying from λ/20 to λ. In addition to strong transmissions in the region below the normal critical angle, very sharp and intense transmission maxima are observed at angles of incidence greatly in excess of the critical angle. These transmissions fall within three clearly defined angular regions: (i) angles between zero and the critical angle for longitudinal waves; (ii) angles between the critical angle for longitudinal waves and the critical angle for transverse waves; and (iii) angles above the critical angle for transverse waves.

Sato, Y., "Study on Surface Waves II; Velocity of Surface Waves Propagated Upon Elastic Plates." Bull. Earthquake Research Inst. Tokyo, v29, pp 223-261, (1951)

In this paper the theory of surface waves propagated upon elastic plates is discussed.

Horizontally polarized shear waves are considered as the simplest model of surface waves, and the resulting dispersion curves are discussed.

The characteristic equation of surface waves formed by P and SV waves is derived and the numerical calculations are performed. The characteristic equation is derived for the case of one side of the plate bounded by liquid, and some numerical calculations are performed. The above equation is expanded in a power series, and it is shown that there are two branches. One of these branches exhibits normal dispersion and the other exhibits abnormal dispersion.

The relationship between phase velocity, group velocity, wave length and period is obtained using the results of a previous article. Particle trajectories are discussed, and it is shown that one of the above mentioned branches represents a bending motion whereas the other branch represents elongation and contraction. Only the real roots of the velocity equations are discussed; however, when the sound velocity in the liquid part is small compared with that of the plate the complex roots must also be considered when dealing with seismograms recorded near the origin.

Sauter, F., "Remarks Concerning the Theory of Vibrations of Thin Elastic Plates" Z. Natursforsch, vA3, pp 548 552, (1948) (In German)

The most natural method of gaining an understanding of vibration phenomena in thin plates, namely that of obtaining rigorous solutions of the basic equations of elasticity, previously has not received extensive treatment in the literature. For thin plates of finite thickness and a given phase velocity, c, this rigorous integration results in a discrete spectrum of frequencies. The author's discussion leads in the limiting case of very thin plates to three distinct laws of dispersion for the three different types of vibrations (shear-, extensional-, and bending vibrations), from which one can easily obtain simplified equations for a first approximation as well as for higher orders of approximation. In particular it becomes easily apparent why the differential equation for the bending vibration of thin plates is of the fourth order in the three-dimensional case.

Schneider, W. E., and Burton, C. J., "Determination of the Elastic Constants of Solids by Ultrasonic Methods," J. Appl. Phys., v20, p 48, (1949)

The application of ultrasonic methods to the determination of the elastic constants of solids is considered in some detail. It is shown that a rotating plate technique in which ultrasonic transmission is plotted as a function of the angle of incidence of the waves allows determination of the velocities of dilation and shear waves in the plate. From these data, Poisson's ratio and the mechanical moduli may be determined. Details of an apparatus for making such measurements are given. The elastic constants of several metals have been measured with this equipment, and the values obtained are shown to be in agreement with previous published data. In addition, measurements of a number of thermo-plastic and thermo-setting resins have been made successfully.

Schoch, A., "Sound Reflection, Sound Refraction, and Sound Diffraction," Berlin, Springer-Verlag, Ergeb. Exakt. Naturwiss., v23, pp 127-234, (1950)

This monograph is a valuable addition to the literature of the theory of sound waves of small amplitude, either in a gas or in an elastic solid. The scope of the work is best indicated by the titles of the chapters, viz.: (I) Introduction; (II) Foundations of the theory; (III) Reflection and refraction of a plane wave at a plane boundary surface; (IV) Free boundary-layer waves along a plane boundary surface; (V) Reflection and refraction of nonplane waves at a plane boundary surface; (VI) Waves in Plates; (VII) Layered media; (VIII) Curved boundary surfaces and diffraction phenomena; (IX) Bibliography. The bibliography should prove most useful, as it contains 144 references, nearly half of them to work done since 1945. While the work is essentially mathematical, it contains many beautiful photographs of experiments illustrating the theory.

Schoch, A., "The Transmission of Ultrasound Through Plates," Nuovo Cim., v7, pp 302-306 (In German)

The transmission by a plate of plane-parallel ultrasonic vibration depends on the frequency, the thickness of the plate, and the angle of incidence. Maximum transmission occurs when

the velocity with which the wave fronts cut the plate equals the phase velocity of any type of wave which can be set up in the plate. Schlieren photographs show how the transmission varies with the plate thickness and angle of incidence. Small damping by the plate gives a strong and broader beam of waves on the rear side of the plate. When very good transmission through the plate occurs, the schlieren photographs show that the reflected wave on the front face disappears.

Schoch, Arnold, "The Transmission of Waves Through Plates," Acustica, v2, pp 1-17 (1952) (In German)

The theory of transmission of sound-plane waves and laterally bounded beams--through plates is given in a form which reveals the connection with the free waves in plates. Cremer's interpretation of total transmission as "coincidence" of the incident wave with a free wave in the plate, certain exceptions from that representation, and the influence of the finite cross section beam are discussed. The conclusions have been examined experimentally on aluminum plates with ultrasonic waves.

Sezawa, K., and Nishimura, G., "Rayleigh-type Waves Propagated Along an Inner Stratum of a Body," Bull. Earthquake Research Inst. Tokyo, v5, pp 85-92, (1928)

Rayleigh-type waves in an inner stratum of an elastic body are classified as symmetrical or asymmetrical when the motion is symmetrical or asymmetrical respectively about the medial plane of the stratum. These waves are dispersive.

The velocity of asymmetric waves is higher than that of symmetric waves for some values of the ratio of wavelength to stratum thickness. The velocities of propagation are limited between the velocities of distortional waves in the matter of which the stratum and outer medium are composed.

The smaller the ratio of the shear modulus of the outer medium to the shear modulus of the stratum, the longer is the range of L/H (ratio of wavelength to thickness of stratum) over which the velocity of propagation is approximately constant.

When the ratio of the shear modulus of the outer medium to the shear modulus of the stratum becomes very small, the velocity

of propagation approaches that of Rayleigh waves on a semi-infinite solid body. The energy of waves of short wavelength accumulates in the vicinity of a weak stratum.

Shamina, O. G.; see Rizinichenko, U. V., and Shamina, O. G., (1957)

Silverman, D.; see Evans, J. J., Hadley, C. F., Eisler, J. D., and Silverman, D. (1954)

Stern, R., see Thurston, G. B., and Stern, R., (1959)

Stonely, R., "Elastic Waves at the Separation of Two Solids," Proc. Roy. Soc. (London), v106, pp 416-429, (1924)

This paper deals with wave motion that is greatest at the surface of separation of two media.

(Considers problem of elastic waves at the surface of separation of two solid media. Waves analogous to Rayleigh waves will propagate. Also considers Love waves in an internal stratus, bounded on both sides by material of other elastic properties.)

Stonely, R., "Rayleigh Waves in a Medium with Two Surface Layers, I.," Monthly Notes Roy. Astr. Soc. Geophys. Suppl., v6, pp 610-615, (1954)

The period equation is obtained in the form of a determinant of the tenth order. If the length of Rayleigh waves propagating in this layered half-space is very small, the determinant reduces to the product of three determinants. They then yield period equations for very short Rayleigh waves along the free surface as well as for similar waves along the two interfaces.

Swenson, George W., Principles of Modern Acoustics, D. Van Nostrand, New York, (1953)

A treatment of theoretical acoustics from the viewpoint of electrical engineering. Chapter headings: Oscillations in Lumped-parameter systems, the Vibrating String, The Vibrating Membrane, Acoustic Waves, Radiators, Acoustic Circuits, Architectural Acoustics, Speech and Hearing, Approximate Methods.

Acoustic waveguide is discussed pp 94-96.

Tash, J. A., "Field Inspection of Boiler Tubes with Ultrasonic Reflectoscope," ASME Trans. v74, pp 201-206, (February, 1952)

Details are given of the ultrasonic reflectoscope and its use in detecting defective small diameter tubing. By means of the ultrasonic shear wave method and the reflectoscope, 90 sections of defective tubing, representing over 1500 feet, were detected and replaced in the convection superheater outlet section of two boilers.

Tubing was 2 inches O. D. × .200" and .220" wall thicknesses.

Thomas, D. A., "Characteristic Impedances for Flexure Waves in Thin Plates," J. Acoust. Soc. Am., v30, p 220, (1958)

Boundary conditions are applied to the general solution of the thin plate wave equation to obtain the solution for outgoing flexure waves in an infinite plate. This solution is used to obtain expressions for a bending moment characteristic impedance and a shear force characteristic impedance. Graphs of these impedances are presented.

Thomson, W. T., "Transmission of Elastic Waves through a Stratified Solid Medium," J. Appl. Physics, v21, pp 89-93, (1950)

The transmission of a plane elastic wave at oblique incidence through a stratified solid medium consisting of any number of parallel plates of different material and thickness is studied theoretically. The matrix method is used to systematize the analysis and to present the equations in a form suitable for computation.

Thomson, W. T., "The Equivalent Circuit for the Transmission of Plane Elastic Waves Through a Plate at Oblique Incidence," J. Appl. Phys., v21, pp 1215-1217 (1950)

Equations are developed for the transmission of a plane elastic wave through a plate at oblique incidence with unequal fluid medium on each side of the plate. The plate transmitting both the dilation and shear wave is reduced to a simple equivalent circuit with impedances which are functions of the incidence angle.

Thurston, G. B., and Stern, R., A Bibliography on Propagation of Sound Thru Plates, Willow Run Labs., University of Michigan, Ann Arbor, Mich. (1959) Office of Naval Research, Contract No. Nonr-1224(24).

A bibliography on propagation of sound through plates. The abstracted material is organized in accordance with a detailed subject outline having five major topics: Transmission thru Plates; Wave Propagation; Properties of Materials; Vibrating Surfaces and Plates; General References. Literature surveyed is principally in the period from 1929 to 1958. Approximately 450 abstracts are given.

Tiffen, R., "Dilational and Distortional Vibrations of Semi-Infinite Solids and Plates," Mathematika, v3, pp 153-163, (1956)

This paper shows that, in the presence of a free plane boundary, special types of purely dilatational and purely distortional vibrations are possible with the presence of Rayleigh waves. Excluding the use of Rayleigh waves, there is only one stable form of distortional plane wave with the displacement vector always parallel to the free surface.

Equations of vibrations of an infinite flat plate are derived with Fourier transform method. In general, it is not possible to consider dilatational and distortional waves separately, except one stable form with displacement vector parallel to the free surfaces.

Timoshenko, S. P., "On the Transverse Vibrations of Bars of Uniform Cross Section," Phil. Mag., v43, p 125, (1922)

An exact solution of the problem is given in the case of a beam of rectangular cross section, of which the breadth is great or small compared with the depth, so that the problem is virtually one of plane strain or of plane stress.

Tolstoy, I., and Usdin, E., "Dispersive Properties of Stratified Elastic and Liquid Media: a Ray Theory," Geophysics, v28, pp 844-870, (1953)

The interference principle of wave guide propagation is applied to the derivation of the period equations for a number of problems involving both elastic and liquid strata. In the case of undamped steady-state propagation in a solid or liquid stratus

two arbitrarily chosen points at the same depth must be indistinguishable on an amplitude basis. This well known principle combined with the knowledge of the reflection coefficients enable us to derive in a few simple steps the period equations for a number of multilayer cases which heretofore had been avoided on account of algebraic difficulties. The period equations obtained by this method exhibit a singular simplicity and unity of form. As a consequence of this one may distinguish formally the M_1 and M_2 waves discovered by Sezawa and understand their physical significance.

Tolstoy, I., and Usdin, E., "Wave Propagation in Elastic Plates: Low and High Mode Dispersion" J. Acoust. Soc. Am., v29, pp 37-42, (1957)

This is an analysis of dispersive properties of elastic plates in vacuo. For low modes there exists conclusive experimental verification of these properties. Model studies show prominent arrivals having the proper spectra and velocities for group velocity maxima and minima corresponding to several symmetric and anti-symmetric modes. In addition, detailed calculations based upon exact formulas predict some new and as yet unconfirmed properties of plates, e.g., negative phase velocities. New results concerning high modes of propagation are also displayed. These modes are of considerable theoretical interest since they belong to the transition region between the domains of validity of the wave and ray theories.

Usdin, E.; see Tolstoy, I., and Usdin, E., (1953)

Usdin, E.; see Tolstoy, I., and Usdin, E., (1957)

Van Valkenburg, H. E.; see Cook, E. G., and Van Valkenburg, H. E., (1954)

Van Valkenburg, H. E.; see Cook, E. G., and Van Valkenburg, H. E., (1955)

Van Valkenburg, H. E., Wilson, R. A., and Olha, W. R., Ultrasonic Inspection Equipment Program, Sperry Products, Inc., Danbury, Conn.; 1 v. incl. illus. tables; Final Engineering Report. 1 Apr. 55- 31 Dec 56 (Technical Report No. TR-065); Contract AF 33(600)29879; AD-143 701; Div. 26/4, 30/3; Unclassified Report.

Research was undertaken to develop improved instrumentation for nondestructive testing of aircraft components by the ultrasonic-reflectoscope method. Investigation of transducers, distance calibration factors, and a digital memory-presentation system are included. Experimental models of the proposed equipment were assembled and evaluated. Transducer development: Piezoelectric materials such as fused titanate ceramics and Li_2SO_4 crystals were studied with emphasis placed on the use of $BaTiO_3$. The work program was divided into 3 main ranges: below 1 mc, 1 to 5 mc, and above 5 mc. Topics which were investigated include lapping, plating, polarizing, damping, and assembly were investigated, and transducer materials and search units were evaluated. Distance Calibration: Amplitude-distance data were obtained for some typical aircraft materials by using the frequencies; search unit sizes, and test hole diameters which are currently used. Distance correction circuitry: Circuits were developed which can be attached to a standard reflectoscope to provide an automatic distance correction for a wide range of test conditions. A practical approach was developed, and suitable equipment was designed, constructed, and tested. Improved flaw-data presentation: A display unit which presents a large bright display which preserves the flaw data until erased was developed by employeng a matrix of 800 small neon lamps similar in principle to an animated billboard (See also AD-118 691) 15 February 1958.

Van Valkenburg, H. E.; see Murdoch, A. M., and Van Valkenburg, H. E., (1959)

Varney, Z. J., and Kleint, R. E., "SNT Airframe Committee Report on Reproducibility of Ultrasonic Inspection Test Results," Nondestructive Testing, v15, n5, pp 290-292, (September-October, 1957)

Report on testing defective 7075 aluminum alloy plate samples; analysis of results compiled from data received from nine companies.

Vernon, J. B.; see Butler, J. B., and Vernon, J. B., (1946)

Volterra, E., and Zachmanoglou, "Longitudinal Waves in An Elastic Plate," Am. Soc. C. E. Proc. 85:33-49 Ja '59

The problem of dispersion of longitudinal waves in an elastic infinite plate is discussed by applying the method of Internal

Constraints and by taking into account second order terms in the equations of constraints. Numerical results obtained by applying this theory are compared with those obtained by applying the exact theory given by Lamb.

Wilson, R. A.; see Van Valkenburg, H. E., Wilson, R. A., and Olha, W. R.

Wilson, R., "Recent Developments in Ultrasonic Inspection of Thin Metallic Sheet or Plate," Sheet Metal Industries v30, n310, pp 146-160, (February, 1953) Discussion n312, pp 308-310, 312, (April, 1953)

New "capacity technique" makes it possible to examine continuity of bond between composite metal plates, standard ultrasonic equipment used, together with suitable assembled transverse wave probes; two main types of Perspex probes indicated; test results, applicability of technique to testing of bonded materials in general. Bibliography.

Worlton, D. C., An Ultrasonic Method For Bond Testing Reactor Fuel Elements. United States Atomic Energy Commission Report No. HW 47017 (Del.) Unclassified, (1956)

Descriptions are given of ultrasonic equipment investigated for determining the integrity of the bonding layer in certain jacketed fuel elements. This method was used to discriminate the size of the unbonded region electronically and also to map the unbonded areas.

An immersion, pulse-echo system is used and the lack of bond is manifested by a stretching of the pulse due to ringing of the unbonded jacket.

Worlton, D. C., "Ultrasonic Testing with Lamb Waves," Nondestructive Testing, v15, n4, pp 218-222, (July-August, 1957)

Application to practical testing problems of Lamb waves which have ability to travel in metal sections that are so thin as to be difficult to resolve by more conventional testing procedures; Lamb waves in test block with built in laminar flaw 0.020 inches below the surface; probe for interior inspection of tubing; Lamb waves in metal strip of different grain structures; possibility of using waves with geometries other than plates.

Worlton, D. C., Lamb Waves at Ultrasonic Frequencies, HW-60662, Hanford Atomic Products Operation, Richland, Washington, (1959)

A theory formulated in 1916 by Horace Lamb predicting that plates may vibrate up to an infinite number of modes is confirmed by a method described. Measurements made on finite plate and curved objects reveal that in each case the wave phase velocity agrees closely with predicted values.

The theory is extended to correlate experimental observations. Equations are developed relating phase velocity to frequency and plate thickness in terms of longitudinal and shear wave velocity. Curves are presented for aluminum, zirconium, stainless steel, brass and uranium.

The distinguishing characteristics of the various modes are discussed in the light of potential nondestructive testing applications. It is shown that interior particles are displaced in elliptical orbits, with vertical motions existing at the surfaces when the wave velocity is $\sqrt{2}$ shear wave velocity, and horizontal surface motions existing for wave velocities equal to longitudinal wave velocity.

Young, R. S.; see Clayton, H. R., and Young, R. S., (1951)

Zachmanoglou, see Volterra E., and Zachmanoglou (1959)

Zeiss, Carl, Instrument for Testing Sheet Metal and Plates (Geraet Zum Pruefen von blechen und platten). Air Technical Intelligence Center, Wright-Patterson Air Force Base, Ohio; (1957); 5 p. incl. illus. (Rept. No. ATIC-275236-A; Trans. No. F-TS-10037/V of rept of VEB Carl Zeiss, Jena, p. 5) AD-147 022, Div. 24/1, 26/4, 30/3, 30/5; Unclassified report

An instrument which used the video sound method for testing sheet metal and plates is described. This method will not only permit detection of possible flaws in the material but will also make the shape and size of the flaws, inhomogeneities, and shrick holes in the material visible. The work piece to be tested is placed into a tub of water and irradiated from one side by an ultrasound generator. The sound pattern of the workpiece is projected onto the water surface through a sound lens; and the relief pattern, created on the surface,

is depicted on a ground glass plate by means of a light-optical Schlieren arrangement. A thin foil is stretched on a slant in the water below the relief pattern. The water wedge forms interference bands of equal thickness which show in the picture only at the sound-permeable spots. By tilting and rotating the foil about the vertical axis, the width and direction of the bands can be arbitrarily varied and adjusted so that the flaws will show distinctly. The entire apparatus was compiled into one structural unit in the new ultrasound camera. Testing can be done in normal daylight with the HBO 200 light source or in a darkroom with a 50-w coil filament lamp. The image on the ground-glass plate can be microfilmed for the testing certificate. The point of a flaw in the material can be marked and later removed. Sheet steel of 10 to 15-mm gage will be penetrated completely by the sound waves, for brass sheets the values are near 10 mm, and for light-metal alloys they are above 20 mm.

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